The Impact of Oil Price Volatility on Welfare in the Kingdom of Saudi Arabia: Implications for Public Investment Decision-making

Axel Pierru and Walid Matar***

ABSTRACT

Since real oil price is positively correlated with real consumption and domestic income in Saudi Arabia, a risk premium needs to be considered when assessing the net present value of oil-related public investment projects. For projects generating additional oil exports, this risk premium quantifies the cost of increased dependence on oil revenues. For projects transforming oil into products whose prices are less correlated with the Saudi economy, it quantifies the benefit from reducing the aggregate risk. The value of this risk premium depends on expectations about future consumption and oil price. By considering alternative assumptions, we show that over a one-year horizon this risk premium could range between 1.3% and 5% of the expected oil-related cash flow, with higher premia for longer planning horizons. We discuss the implications of these calculations for energy-related public projects in Saudi Arabia and, more generally, for public decision-making in resource-rich countries.

Keywords: Risk premium, Oil, Public investment, NPV, Domestic income, Saudi Arabia.

http://dx.doi.org/10.5547/01956574.35.2.5

1. INTRODUCTION

Maximizing economic welfare is a primary objective of policymakers worldwide. However, under the reasonable premise that agents are risk averse, the uncertainty surrounding the economic growth rate has a social cost, usually determined as the loss of welfare that a representative agent is willing to incur to get rid of fluctuations in his consumption or income. Though this cost may be negligible in certain economies, this may not be the case for countries that rely on commodity exports revenue. Since the Kingdom of Saudi Arabia is the world's largest oil exporter, a considerable portion of its gross domestic income and government revenues depends on the crude oil price. As a consequence, Saudi domestic income and aggregate consumption are likely to be variable throughout time and significantly correlated with the crude oil price. In recent years, oil-related exports have on average represented around half of the Saudi nominal Gross Domestic Product (GDP). Over the last two decades, the standard deviation of changes in the annual average price of Arabian Light crude oil is 25%. A one-standard-deviation shock to the oil price therefore

* Corresponding author. King Abdullah Petroleum Studies and Research Center (KAPSARC), P.O. Box 88550, Riyadh 11672, Saudi Arabia. Phone: 966-536002497, E-mail: axel.pierru@kapsarc.org.

** King Abdullah Petroleum Studies and Research Center (KAPSARC), P.O. Box 88550, Riyadh 11672, Saudi Arabia. Phone: 966-536004253, E-mail: walid.matar@kapsarc.org.

represents an income shock equivalent to 12.5% of Saudi GDP, which is high relative to the GDP volatility in most countries.

For public investment decision-making in Saudi Arabia, this raises the question of the risk premium associated with the crude oil price. In other words, when assessing a public project's net present value, by which amount should its expected oil-related cash flows be adjusted? This adjustment quantifies the social cost—or benefit—generated by the correlation of these cash flows with economic growth. Considering this risk premium may affect the decision making pertaining to energy-related public projects in Saudi Arabia.

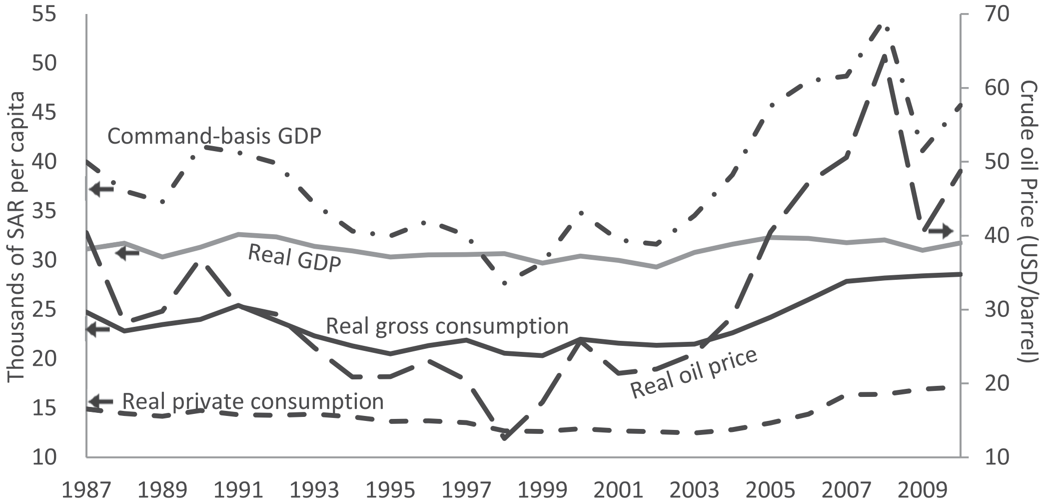
The sustainability of the current path of Saudi domestic oil consumption has recently been questioned (e.g., Gately et al. (2012)). To curb the growth in domestic oil demand and thus free additional oil for export, various options are currently being considered by Saudi authorities, especially the diversification of the Saudi energy mix and investments in energy efficiency. In particular, the Saudi power sector relies almost exclusively on oil and natural gas. Developing new energy sources for power generation would therefore help preserve oil exports, as in some regions of the Kingdom the marginal power generation technology is based on the combustion of oil products. Saudi Arabia thus announced[1] a solar power generation capacity target of 41 GW by 2032 in an attempt to decrease oil consumption in the electricity sector. The Kingdom is also exploring the possibility of introducing nuclear power capacity in its energy mix over the coming decades. Cooperation and research agreements have thus been signed with France, South Korea and China to advance this effort. In addition, the national oil company Saudi Aramco is developing oil and gas fields in the eastern and northern provinces to secure additional future production. All these projects, whose profitability is ultimately driven by increased oil exports, may potentially be negatively impacted by this risk premium.

Alternatively, any project turning oil into a product whose price is less correlated with the Saudi economy should benefit from an adjustment quantifying the gain from increased risk diversification (i.e., a negative risk premium). The ongoing move towards downstream chemicals requiring crude-oil-based feedstock may provide examples of such diversifying projects. More generally, the Kingdom is committed to use its oil resources to diversify its economy and reduce its dependence on oil revenues through physical and social investments.

Quantifying the risk premium will provide a more accurate valuation of all public energy-related projects in the Kingdom. The literature dealing with macroeconomic fluctuations in Saudi Arabia (e.g., Rosser and Sheehan (1995), Dibooglu and Aleisa (2004), Mehrara and Oskoui (2006)) has not addressed this issue. The international literature in general offers very few empirical assessments of risk premia to consider when valuing public investment projects. For instance, Van Ewjik and Tang (2003) discuss the value of risk premia for public projects in the Netherlands; Gollier et al. (2011) discuss this issue in the French context. For the European Union, Durand-Lasserve et al. (2010) compute a risk premium associated with the CO_2 price that is consistent with the optimum of their global general equilibrium model. However, to the best of our knowledge, there is no empirical assessment of the risk premium that would be associated with a commodity exported by a resource-rich country. This paper proposes an empirical assessment of the risk premium that could be attributed to crude oil price by Saudi authorities. A significant part of the considerations and methodology presented here could however be transposed to other resource-rich countries, like other OPEC members.

1. The proposals for solar and nuclear energy were announced by the King Abdullah City for Atomic and Renewable Energy at the Fourth Saudi Solar Energy Forum (2012).

Figure 1: Real Oil Price and Saudi GDP, Command-basis GDP and Consumptions (1987–2010)



The next section establishes a simple framework for public investment decision making. Subsection 2.1 examines how past Saudi aggregate consumption and domestic income have been volatile and correlated with crude oil price. In this respect, to become a measure of domestic income, the real GDP has to be adjusted for improvements (or deteriorations) in the Kingdom's terms of trade. We suggest using a command-basis GDP. Subsection 2.2 resumes Gollier's (2007) derivation of the risk premium formula and discusses practical issues for the assessment of this risk premium, as well as related questions on public investment decision-making in the Kingdom. Subsection 2.3 makes some assumptions that will serve to compute the risk premium. Through a simple assessment of the social cost of macroeconomic risks, Section three proposes a first calibration of the short-term (i.e., over a one-year horizon) risk premium associated with oil price. Section four uses two alternative approaches to provide estimates of this risk premium in the long run (i.e., over longer planning horizons): an approach based on a restrictive joint log-normal assumption and another based on cointegration analysis. The last section discusses the implications of these calculations for energy-related public projects in Saudi Arabia and, more generally, for public decision-making in resource-rich countries.

2. A SIMPLE FRAMEWORK FOR PUBLIC INVESTMENT DECISION-MAKING

2.1 Real Oil Price, Saudi Consumption and Domestic Income: A First View

Figure 1 shows the Kingdom's per-capita Gross Domestic Product (GDP), command-basis GDP, private and gross consumptions, as well as the oil price, all series being expressed in real terms with 1999 as the base year. The data used are provided by Table A6.

A country's real gross domestic income measures the purchasing power of the total incomes generated by its domestic production. The Saudi real GDP, which is computed by the Saudi authorities at constant 1999 prices, ignores the changes in the relative prices of exports and imports and, therefore, likely underestimates the actual fluctuations in the Kingdom's domestic income. As the world's largest oil exporter, the Kingdom has the nominal value of its exports driven by the volatile crude oil price, while a large portion of its imports consists of manufactured products, which have stickier prices. Over short horizons, the changes in the relative prices of exported oil and imported goods may have a strong impact on the Saudi domestic income; an impact not necessarily reflected in the real GDP. For instance, between 1998 and 1999 a decrease in the volume of Saudi oil exports, along with a simultaneous increase in the nominal price of oil relative to that of imported goods, has led to a decrease in the Saudi real GDP but an increase in the Saudi real domestic income.

To be interpreted as real domestic income, real GDP has to therefore be adjusted for improvements (or deteriorations) in the Kingdom's terms of trade. We suggest using the Kingdom's command-basis GDP per capita as a measure of the Saudi real domestic income. As discussed by Kohli (2004), instead of deflating nominal imports by the price of imports and nominal exports by the price of exports as in the case of real GDP, the net exports are deflated by the import price deflator. The rationale for this approach is that to quantify real income, what matters is not the quantity of goods and services that are exported, but rather the quantity of imports that is made possible through these exports. The command-basis GDP values used here result from our own[2] calculations. For the sake of simplicity, we adjusted the real GDP by considering that all exports were oil-related, since non-oil exports represent only a small fraction of Saudi exports (12.7% on average between 2007 and 2010); half of them being made of petrochemical products whose prices are reasonably well-correlated with oil price. Since the geographical origin of the Saudi imports is relatively well-diversified, with a large share of consumer goods, the World Bank's world consumer price index (world CPI) is used as a proxy for the import price deflator. The adjustment for year t is therefore the difference between the nominal exports deflated by the world CPI and the product of the nominal exports by the ratio of the oil price in 1999 over the nominal oil price in year t.

In this paper, the real price of crude oil, given[3] in 1999 USD per barrel, is defined as the nominal price of the Arabian Light deflated by the World Bank's world CPI. The gross consumption is the sum of private and government consumptions. The government consumption, whose share in the Kingdom's economy is relatively important, includes non-durable public goods. In this paper, the gross consumption is therefore considered as a relevant indicator for consumption. Data in real terms are only available from 1998 to 2010. Since during this period the deflator used is almost indistinguishable from the Saudi Arabian cost of living index, we have used this index to deflate the pre-1998 data.

The five series appear to be non-stationary. Table A1 shows that all series are integrated of order one since the null hypothesis of a unit root cannot be rejected whereas for the differenced series, this hypothesis can be rejected at 5% significance; Table A2 shows that the series of annual relative percentage changes are stationary.

Table 1 gives the coefficients of correlation between the relative percentage changes in real gross/private consumption and real GDP, command-basis GDP or real crude oil price. The positive coefficient of correlation between gross consumption and command-basis GDP is significant at a 1% level, and the coefficient of correlation between private consumption and command-

2. To benchmark our calculation, we have also used a real income per capita measure adjusted for terms of trade provided by Penn World Tables (RGDPTT in PWT 7.0, defined as per-capita PPP Converted Gross Domestic Income at 2005 constant prices). Over the period considered, the annual relative change in this measure and that in our command-basis GDP exhibit a high correlation coefficient of 87% (significant at a 1% level). Both real domestic income series also display similar volatilities of 10% and 11% (whereas the per-capita real GDP exhibits a volatility of 2.6%). They seem therefore consistent.

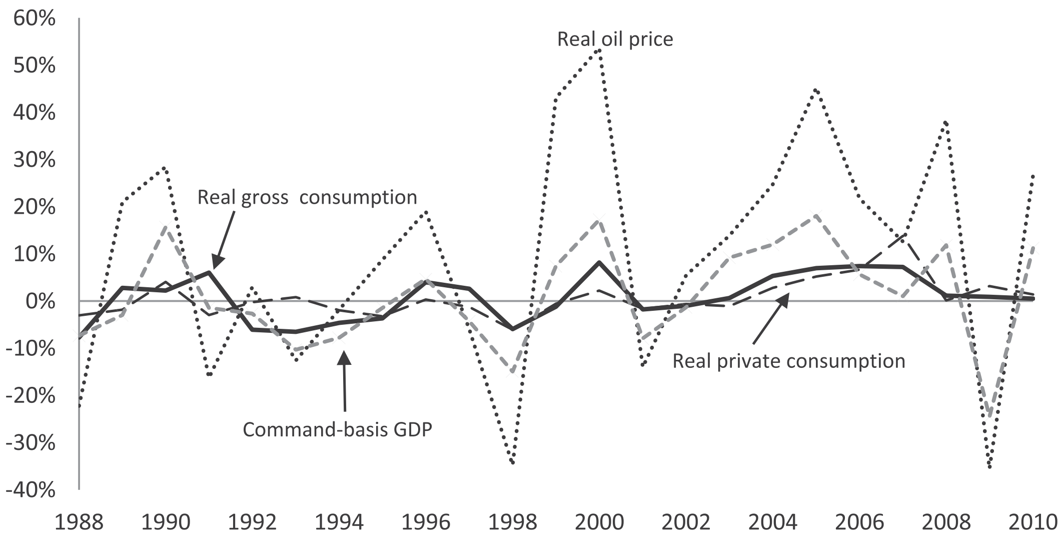
3. It has to be noted that the exchange rate has remained constant at 3.75 Saudi riyals per U.S. dollar since mid-1986.

Table 1: Coefficients[a] of Correlation between Relative Percentage Changes (1988–2010)

	Real GDP per capita	Command-basis GDP per capita	Real crude oil price
Real gross consumption per capita	33.4%	56.9%***	55.1%***
Real private consumption per capita	1.59%	34.8%	39.0%*

[a] The confidence intervals are calculated by first applying a Fisher transformation. Here and in the remainder of the paper, ***, **, and * denote statistical significance at 1%, 5%, and 10% levels, respectively.

Figure 2: Relative Changes in Real Oil Price (dotted line) and in—real per-capita—Gross Consumption (solid line), Private Consumption (long dashes), and Command-basis GDP (short dashes)



basis GDP is significant at an 11% level. By suggesting that changes in terms of trade translate into changes in consumption, this substantiates the idea that the command-basis GDP is more closely associated with the Saudi society's utility curve and more in line with the Kingdom's real private and government consumptions than the real GDP. By regressing real private and government consumptions on real GDP and trading gains over the period 2003–2007, MacDonald (2010) obtains similar results for OECD resource-rich nations, like Norway or Australia. She finds that real consumption advanced more than real production in these countries which have experienced large terms-of-trade improvements.

Figure 2 illustrates the coefficients of correlation between the relative percentage changes for the period of 1988 to 2010 for the crude oil price, the per capita gross and private consumptions, and the per capita command-basis GDP.

For public investment decision-making, since Saudi consumption and income measures appear to be positively correlated with crude oil price, risk premiums have to be considered when calculating an oil-related project's net present value (NPV). In the next subsection, the standard formula of the risk premium to consider is derived as in Gollier (2007) and discussed in the context of our paper.

2.2 Standard Formula of the Risk Premium

Following a classical approach, we consider that the expected total utility, which is defined as the sum of expected utilities of per-capita consumption for current and future populations, is the welfare measure maximized by the Saudi authorities. Let C_t denote the optimal consumption per capita in year t, with t ranging from zero to infinity. Only C_0 is deterministic, whereas $(C_1, C_2, \ldots)$ are exogenous random variables whose distributions, conditional on the information available at $t = 0$, are assumed to be known. The expected total utility is written as follows:

$$\sum_{t=0} e^{-\rho t} E(l_t u(C_t))$$

Where $u()$ is the utility function, l_t is the size of the Saudi population in year t, and ρ is the rate of time preference (used to discount utility).

Let us now consider a public oil-related investment project that in year t ($t = 0,1,\ldots\infty$) would generate the (uncertain) cash flow $F_t + b_t P_t$, where P_t is the oil price, b_t is a coefficient representing the number of barrels freed for export (or consumed if $b_t < 0$), and F_t may be a capital expenditure, an operating expense or even a revenue. With the exception of $F_0 + b_0 P_0$, all these future cash flows are uncertain. This investment project is profitable if it increases the welfare of the Saudi society:

$$\sum_{t=0} e^{-\rho t} E\left(l_t u\left(C_t + \frac{F_t + b_t P_t}{l_t} \right)\right) \geq \sum_{t=0} e^{-\rho t} E(l_t u(C_t)) \tag{1}$$

With a first-order Taylor expansion in C_t and simple manipulation (that is valid as long as the project size does not exceed a small fraction of the Saudi gross domestic income), (1) becomes:

$$\sum_{t=0} e^{-\rho t} \frac{E(u'(C_t))}{u'(C_0)} \left(E(F_t + b_t P_t) + cov\left(F_t + b_t P_t, \frac{u'(C_t)}{E(u'(C_t))} \right)\right) \geq 0 \tag{2}$$

By setting $r_t = \rho - \frac{1}{t} ln\left(\frac{E(u'(C_t))}{u'(C_0)} \right)$, we introduce the public discount rate r_t which is independent from the project under study. This discount rate represents a trade-off between immediate marginal utility and future expected marginal utility. Estimating the value of the discount rate that could be used by Saudi public authorities is not in the scope of this paper. This value primarily depends on their expectations about future economic growth. However, for the sake of illustration, we can apply Gollier's (2007) generalized form[4] of the Ramsey rule. For instance, by setting $\rho = 0$

4. This formula, derived under certain assumptions, yields a constant discount rate r_t defined as the sum of ρ, a wealth effect and a precautionary effect. The wealth effect is equal to the relative risk aversion times the expected consumption growth rate (i.e., the more future generations will consume, the higher the discount rate). The precautionary effect is equal to minus half the product of the variance of this growth rate, the relative risk aversion, and one plus the relative risk aversion (i.e., the more uncertain the future consumption, the lower the discount rate).

for intergenerational equity and considering a relative risk aversion coefficient ranging between one and three (this issue is discussed in Subsection 2.3), using historical gross consumption we obtain very low values for the real social discount rate, ranging from 0.5% to 0.8%.

We can rewrite (2) as follows:

$$\sum_{t=0} e^{-r_t t}\left(E(F_t + b_t P_t) + cov\left(F_t + b_t P_t, \frac{u'(C_t)}{E(u'(C_t))}\right)\right) \geq 0 \tag{3}$$

As emphasized by Gollier (2007), to implement this approach, the following first-order approximation is usually made:

$$E(u'(C_t)) \cong E(u'(E(C_t)) + (C_t - E(C_t))u''(E(C_t))) = u'(E(C_t))$$

We can therefore make the following approximation:

$$cov\left(F_t + b_t P_t, \frac{u'(C_t)}{E(u'(C_t))}\right) \cong -\alpha \operatorname{cov}\left(F_t + b_t P_t, \frac{C_t}{E(C_t)}\right) \tag{4}$$

Where α denotes the coefficient of relative risk aversion at the expected consumption, with $\alpha = -E(C_t)\dfrac{u''(E(C_t))}{u'(E(C_t))}$.

By combining (3) and (4), we obtain the standard condition in public economics that to be profitable the project must have a non-negative NPV:

$$F_0 + b_0 P_0 + \sum_{t=1} e^{-r_t t}\left(E(F_t) - \alpha \operatorname{cov}\left(F_t, \frac{C_t}{E(C_t)}\right) + E(b_t P_t) - \alpha \operatorname{cov}\left(b_t P_t, \frac{C_t}{E(C_t)}\right)\right) \geq 0 \tag{5}$$

Every cash flow therefore impacts the project's NPV through its expected value and a risk premium proportional to its covariance with $\dfrac{C_t}{E(C_t)}$. This risk premium is positive if the cash flow is positively correlated with the Saudi economic activity, since receiving this cash flow then increases the global risk borne by the Saudi society.

So far we made no specific assumption about b_t. However, the operating costs of projects like the development of solar or nuclear power generation capacities are relatively low (compared to oil or gas-based power generation), which implies that the capacity envisioned by Saudi authorities should be used. The same reasoning can be held for investments in energy efficiency. We may therefore consider that b_t is deterministic, or, at least, not correlated with the economic activity or the crude oil price. For projects for which this assumption cannot be made, additional elements, like the correlation between the demand addressed to the project and the economic activity, would have to be considered.

The formula of the consumption risk premium associated with one barrel of oil in year t is consequently: $\alpha \operatorname{cov}\left(P_t, \dfrac{C_t}{E(C_t)}\right)$.

As previously shown, the oil price is positively correlated with all measures of Saudi consumption or income. Therefore, all projects whose profitability is ultimately driven by additional Saudi oil exports are likely to increase the macroeconomic risk borne by the Saudi society. On the contrary, any project transforming oil (i.e., $b_t < 0$) into a product whose price is less correlated with the Saudi economy will have a negative risk premium, i.e. a positive cash flow, quantifying the benefit from risk diversification. This negative risk premium can be viewed as an insurance value, since undertaking the project reduces the aggregate risk in the economy.

2.3 Scenarios for Future Consumption and Relative Risk Aversion

When assessing the profitability of an oil-related project, public authorities may consequently take into account a risk premium proportional to $\alpha \operatorname{cov}\left(P_t, \frac{C_t}{E(C_t)}\right)$. This term, which does not depend on the project under study, needs to be determined at the level of the Saudi economy.

All equations in Section 2.2 are derived from marginal changes around a future optimal stream of consumption C_t that is currently unknown. It might be argued that the income of the representative agent is uncertain (as it is subject to external shocks, like those on the oil price) and that, to a certain extent, this uncertainty spills over to consumption (and results in consumption fluctuations) through the arbitrage between consumption and saving. The fluctuations in future consumption will therefore depend on government's saving policy. In Saudi Arabia, on a historical basis, the path followed by consumption (with historical volatilities of 4% and 4.9% for private and gross consumptions respectively) is much smoother than that followed by domestic income (with a volatility of 11% for command-basis GDP), as saving has been used as a buffer against oil-price shocks. Calibrating the risk premium on historical real consumption thus corresponds to a 'moderate-volatility scenario' for future consumption.

However, in the future, government and private consumption might adjust to changes in income in a different way. In this respect, making the theoretical assumption that saving would represent a constant proportion of real domestic income generates a 'high-volatility scenario' for future consumption. We do not pretend to provide any foundation or credibility for this scenario, we just consider that it can serve to define an upper bound for the risk premium. The risk premium can then be assessed by computing the covariance between the real oil price and command-basis GDP. This risk premium will be higher than that determined using historical consumption. In this paper, we consequently calibrate the risk premium on both historical real domestic income (to obtain an upper bound) and historical real consumption (to obtain a lower bound).

Furthermore, it should be noted that the risk premium is proportional to the coefficient of relative risk aversion. As mentioned by Lucas (2003), estimates of the parameter α in use in macroeconomics and public finance applications range from 1 to 4. As far as we know, there is not any specific study addressing the value of α for the Kingdom of Saudi Arabia. However, when the risk relates to flows of costs or benefits, a relative risk aversion coefficient of two has often been used in the literature (e.g. Chetty (2006), Gollier (2007), Hall and Jones (2007), Dasgupta (2008), Weitzman (2009)). In a report commissioned by the French government, Gollier et al. (2010) also recommend using a coefficient of two for public decision purposes. This value will consequently play a central role in the numerical illustrations performed in this paper. However, sensitivity analyses around this value will also be provided.

3. CALIBRATION OF THE SHORT-TERM RISK PREMIUM ASSOCIATED WITH OIL PRICE

In a very simple way suggested by Gollier (2001), we first assess the social cost of the volatility of the Saudi domestic income over one year. We consider that all inhabitants of the

Kingdom can be represented by a single representative agent. The risk faced by this representative agent is then measured by the uncertainty surrounding the domestic income per capita. The social cost of the macroeconomic risk can be measured by the reduction in the expected income that the risk-averse representative agent would be ready to pay to eliminate the income volatility. To compute the cost, we therefore need to determine the certain income that generates the same level of utility as the volatile income I_1. Expressed as a fraction of the expected income, the cost of macroeconomic risk, denoted as λ, is consequently defined by the following equation:

$$E(u(I_1)) = u((1-\lambda)E(I_1)) \tag{6}$$

By Taylor expansion[5] in $E(I_1)$, we have:

$$u(I_1) \cong u(E(I_1)) + (I_1 - E(I_1))u'(E(I_1)) + \frac{(I_1 - E(I_1))^2}{2}u''(E(I_1))$$

$$u((1-\lambda)E(I_1)) \cong uE(I_1)) - \lambda E(I_1)u'(E(I_1))$$

By replacing the left-hand and right-hand sides of (6) with the expanded forms, we have:[6]

$$\lambda \cong \frac{\alpha\, var(I_1)}{2E^2(I_1)} \tag{7}$$

Where α denotes the coefficient of relative risk aversion at the expected income.

For the measure of the Saudi per-capita domestic income under consideration, let us assume that next year the growth rate of this income will be an outcome of the random variable G, with $E(G)$ (the expected value of G) and $var(G)$ (the variance of G) respectively given by the historical mean and variance of the corresponding stationary time series. Let I_0 be the known income in the current year, with consequently: $I_1 = (1 + G)I_0$. From (7), we have:

$$\lambda \cong \frac{\alpha\, var(G)}{2(1 + E(G))^2} \tag{8}$$

Table 2 gives the historical mean and variance of the relative percentage change for the per-capita real GDP and command-basis GDP. Not surprisingly, the growth rate of the command-basis GDP appears to be much more volatile than that of the real GDP.

For the sake of illustration, by considering $\alpha = 2$ and using figures in Table 2, Formula (8) yields[7] an estimate of short-term (one-year-ahead horizon) cost of macroeconomic risks equal to

5. For each income growth rate series, a normal distribution is not rejected by the Jarque-Bera test. This supports the validity of a second-order approximation in the left-hand side of (6), since $E((I_1 - E(I_1))^3)$ is proportional to the third moment of the growth rate variable G (introduced below).

6. Consistent with formula (5), the cost $\lambda E(I_1)$ can be considered as a cumulative risk premium resulting from increased exposure to the systematic risk: $\int_0^1 \alpha\, cov\left(sI_1, \frac{sI_1}{E(sI_1)}\right)ds = \frac{\alpha\, var(I_1)}{2E(I_1)} = \lambda E(I_1)$.

7. For each income measure, we have also determined the cost λ by assuming that all growth rates realized in the past may occur with equal probability next year (which defines a probability distribution for G). For $\alpha = 2$, this amounts to numerically solving $E((1 + G)^{-1}) = ((1-\lambda)(1 + E(G)))^{-1}$. The cost λ thus obtained is very close, which confirms the robustness of the approximations made in this section.

Table 2: Mean and Variance of Annual Growth Rates (1988–2010)

	Growth rate of real GDP per capita	Growth rate of command-basis GDP per capita
Mean ($E(G)$)	0.12%	1.16%
Variance ($var(G)$)	0.07%	1.17%

1.14% when the command-basis GDP is the selected income measure and only 0.07% when the real GDP is the selected measure. For a coefficient of relative risk aversion equal to unity, the cost is only 0.035% when the real GDP is the income measure considered, which is in line with results obtained for other countries. Using annual U.S. data for the period of 1947–2001, for example, Lucas (2003) shows that the welfare annually gained by eliminating all consumption fluctuations around an exponential consumption path would be about one-twentieth of one percent of consumption. For the Kingdom, this cost is greater by more than an order of magnitude when the income measure I used is the command-basis GDP.

The oil price risk premium $\alpha cov\left(P_1, \frac{I_1}{E(I_1)}\right)$ over one year (i.e., the "short-term" risk premium) can now be calibrated, based on the value of λ determined with the command-basis GDP. In Appendix B, under simplifying assumptions, we calibrate the risk premium that should be considered in 2010 from a 2009 perspective and the risk premium that should be considered in 2009 from a 2008 perspective, for a relative risk aversion coefficient of 2. Making these two calibrations serves to test the sensitivity of the risk premium to the oil price. As a result, from a 2009 perspective, the risk premium in 2010 would have amounted to 3.72 dollars per barrel (i.e., 6.1 percentage points of the oil price realized in 2009). With similar calculations, from the 2008 perspective, in 2008 dollars the risk premium in 2009 would have been five dollars per barrel (i.e., 5.3 percentage points of the oil price realized in 2008).

These calibrations suggest that for the Saudi economy, the short-term oil price risk premium may exceed 5% of the oil price when a relative risk aversion coefficient of two is considered. It represents almost 3% of the oil price when the relative risk aversion considered is only unity, and may exceed 8% of the price for a relative risk aversion of 3. This risk premium, however, depends on the covariance of two non-stationary variables and is therefore likely to increase with respect to the time horizon considered. The next section proposes two alternative econometric assessments of the 'long-term' risk premium.

4. ASSESSMENT OF THE LONG-RUN OIL PRICE RISK PREMIUM

As shown by Table A1, all series are integrated of order 1, which suggests that the covariance between oil price and Saudi income or consumption increases throughout time. As a result, the farther in the future the expected oil-related cash flow is located, the greater should be the risk premium to consider. A first and straightforward evaluation procedure of the risk premium in the long run derives from the restrictive assumption that both times series are jointly lognormal. A second procedure consists in testing for the existence of cointegration relationships between oil price and variables specific to Saudi Arabia. These two procedures are successively applied in the following subsections.

Table 3: Normality Test Results for Changes in Log of Saudi Income and Consumptions

	Command-basis GDP per capita	Real gross consumption per capita	Real private consumption per capita
Jarque-Bera test value	1.39	1.27	10.77***

Critical values: 4.61 (10%); 5.99 (5%); 9.21 (1%)

4.1 The Joint Lognormal Assumption

We assume here that the real oil price, the gross consumption per capita and the command-basis GDP per capita follow a geometric Brownian motion. This assumption is supported by the fact that the logarithms of these three series are first-order integrated (as shown by Table A1) and that the Jarque-Bera test does not reject a normal distribution for the corresponding series of changes in log (as shown by Table 3). We cannot assume that the private consumption follows a geometric Brownian motion since Table 3 shows rejection of the normal distribution for log change in this series at 1% significance level.

In addition to this geometric Brownian assumption, let us hypothesize here that the utility function exhibits a constant relative risk aversion coefficient . Under these specific assumptions, the exact formula of the risk-premium can be derived from (3), as shown by Gollier (2012).

Let us first consider the command-basis GDP, in order to derive an upper bound for the risk premium. The risk premium in year t then amounts to the fraction $1-e^{-\alpha \times 2.4\% \times t}$ of the expected oil price, where 2.4% is the estimated covariance between the changes in log of real oil price and the changes in log of command-basis GDP. For a relative risk aversion of 2, the short-term risk premium is therefore equal to 4.8% of the expected oil price, which is consistent with the calibration achieved in Section 3. This risk premium represents half the expected oil price in a 15-year horizon, and three quarters of the expected price over 30 years. Over an infinite horizon, the risk premium tends towards the expected oil price. It is noteworthy that subtracting this risk premium from the expected oil price is equivalent to discounting the expected oil price at a rate that includes a 4.8% risk premium. If the risk-free discount rate used were for instance 1% (see Section 2.2), the real oil-price-related cash flows would have to be discounted at a rate of almost 6%. Note that this amounts to assuming an oil-price consumption beta of 2, computed as the estimated covariance (2.4%) divided by the variance of the change in the log of command-basis GDP per capita (1.2%).

If we now consider the real gross consumption per capita, the calculated risk premium in year t is much lower and equal to the fraction $1-e^{-\alpha \times 0.64\% \times t}$ of the expected oil price (with a corresponding consumption beta of 2.66). For a relative risk aversion of 2, over one year this risk premium amounts to 1.27% of the expected oil price. It amounts to 12% of the expected oil price in a 10-year horizon.

4.2 Estimation of Cointegration Relationships

To apply a more general approach, we test for cointegration between oil price and each Saudi macroeconomic variable. The results of the Johansen cointegration tests are shown in Table 4, and the corresponding estimated bivariate vector error correction (VEC) models are given by Tables A3 to A5.

Table 4: Johansen Cointegration Tests between Each Saudi consumption and Income Variable and Real Crude Oil Price (1987–2010)

		Trace		*Maximum eigenvalue*	
	Null hypothesis	Test statistic	Significance level	Test statistic	Significance level
Real gross consumption per capita (one lag)	r = 0	20.31	**	17.52	**
	r ≤ 1	2.79	—	2.79	—
Real private consumption per capita (two lags)	r = 0	32.07	***	19.03	**
	r ≤ 1	13.02	***	13.02	***
Real command-basis GDP per capita (two lags, linear trend)	r = 0	17.39	**	16.65	**
	r ≤ 1	0.74	—	0.74	—

In all cases, the null hypothesis of absence of cointegration can be rejected. For private consumption, the absence of a second cointegrating relationship can also be rejected. This would however imply that neither oil price nor private consumption have a unit root, which seems unlikely. This might result from the shortness of the period considered, since for a model with one lag the tests indicate only one cointegrating relationship at 5% significance level.

All models have been selected under the condition that the residuals behave well, with, at 5% significance level, no rejection of the following null hypotheses: normal distribution, absence of serial correlation, and homoscedasticity. For gross consumption and oil price, the one-lag model minimizes both Schwarz and Akaike information criteria. For command-basis GDP and oil price, the selected[8] two-lag model minimizes the Schwarz information criterion. For private consumption, we selected[9] the two-lag model by considering that the oil price should be weakly exogenous. As a result, in each model, the adjustment coefficient estimated for the Saudi-variable equation is significant and of the expected sign.

Let us consider any of the three Saudi macroeconomic variables, for instance the real gross consumption per capita, C_t, and write the identified long-run equilibrium between this variable and the real crude oil price:

$$C_t - k_0 - k_1 P_t = \varepsilon_t$$

Where k_0 and k_1 are the coefficients estimated in the cointegration relationship, and ε_t is the stationary disequilibrium term. Table 5 gives the values of k_0 and k_1 estimated in each vector error correction model.

Hence, we have:

$$cov(P_t, C_t) = cov(P_t, k_0 + k_1 P_t + \varepsilon_t) = k_1\, var(P_t) + cov(P_t, \varepsilon_t)$$

8. Since initially none of the two adjustment coefficients was significant, we restricted the adjustment coefficient in the oil price vector to zero.

9. A one-lag model yields a slightly lower Akaike information criterion, but with a significant adjustment coefficient in the oil price vector, whereas weak exogeneity of oil price seems more plausible.

Table 5: Coefficients Estimated for Each Long-run Equilibrium

	k_0	k_1
Real gross consumption per capita	16,697	222.86
Real private consumption per capita	8,177	206.34
Command-basis GDP per capita	20,089	587.46

Table 6: Risk Premium over a One-year Horizon, in Percentage of the Expected Oil Price

Relative risk aversion (α)	Real private consumption per capita	Real gross consumption per capita	Command-basis GDP per capita
1	1.15%	0.85%	1.21%
2	2.29%	1.69%	2.42%
3	3.44%	2.54%	3.63%

With:

$$\lim_{t \to \infty}\left(\frac{cov(P_t, C_t)}{var(P_t)}\right) = k_1$$

Consequently, to implement the approach, we may consider that in the long-run:

$$\alpha\, cov\left(P_t, \frac{C_t}{E(C_t)}\right) = \frac{\alpha k_1\, var(P_t)}{k_0 + k_1 E(P_t)} \tag{9}$$

According to (9), a growth in the oil price variance throughout time induces a proportionate growth in the risk premium. Therefore, the risk premium that the Saudi authorities should consider in the long-run depends on their view of the future of the oil price.

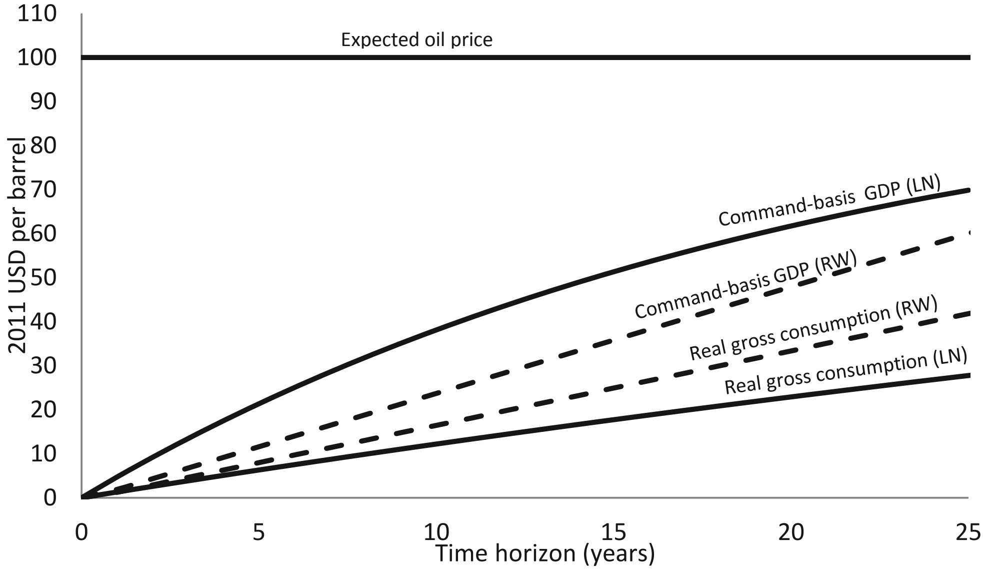
For the sake of illustration, let us assume that the real oil price follows an arithmetic random walk with a variance of annual price increments (estimated on the historical time series) equal to 68.6. In addition, the expected value of the real oil price is assumed to remain constant and equal to 100 in 2011 dollars. Using (9), Table 6 provides[10] the corresponding risk-premium over a one-year horizon for different values of the relative risk aversion coefficient. Under the arithmetic random-walk assumption made here the variance of oil price, and therefore the risk premium, is proportional to the time-horizon considered.

For a relative risk aversion coefficient of 2, the risk premium over one year is thus equal to 1.69% of the expected crude-oil price when gross consumption is used, 2.29% when private consumption is used, and 2.42% when command-basis GDP is used. Over a 10-year horizon, this risk premium consequently lies between 16.9% and 24.2% of the expected oil price.

To compare the alternative risk premium calibrations achieved in Sections 4.1 and 4.2, from a 2011 perspective let us again adopt the view that, in 2011 dollars, the expected value of the

10. These short-term figures do not incorporate the disequilibrium-related term. Incorporating this term, estimated as the historical covariance between the stationary variable ε_t and the non-stationary variable P_t, gives a short-run risk premium of 1.88% for command-basis GDP and 1.20% for gross consumption.

Figure 3: Risk Premium with Respect to Time (LN: joint lognormal assumption, RW: random walk assumption) for a Relative Risk Aversion of 2



real oil price for the subsequent years remains constant at 100 dollars per barrel. Figure 3 illustrates the upper and lower risk premium curves, based on historical data, for both joint lognormal and random walk assumptions, under the assumption that the relative risk aversion coefficient is 2.

5. CONCLUSIONS AND IMPLICATIONS FOR PUBLIC ENERGY-RELATED DECISIONS

A considerable portion of the Saudi domestic income depends on volatile oil revenues. In presence of risk aversion, this dependence has a social cost, which requires considering a risk premium when valuing an energy-related public investment project. This risk premium has to be subtracted from expected oil-price-related cash flows when assessing the project's net present value. The literature in general does not provide any thorough estimate of the risk premium associated with the price of an exported commodity for public decision-making in resource-rich countries.

In this paper, we attempt to quantify the risk premium associated with the crude oil price for public investment decision-making in Saudi Arabia. As the magnitude of the risk premium depends on future consumption patterns, possible upper and lower bounds have been determined for this risk premium, by considering the historical per-capita gross domestic income (defined as the command-basis GDP) and real gross consumption.

When a relative risk aversion coefficient of two is considered, over a one-year horizon this risk premium may lie between 1.3% and 4.8% of the expected oil price. It is likely to increase for longer planning horizons. In other words, the further in the future the expected oil-related cash flow, the higher is the risk premium to consider.

For a practical illustration, let us consider a 20-year stream of cash flows derived from the sale of an oil barrel at market price every year. Let us also assume that the expected real oil price remains constant at 100 dollars per barrel for the next 20 year. Ignoring the risk premium would imply determining the present value of this stream of cash flows as the expected oil price multiplied by the sum of discount factors. Taking into account the risk premium implies subtracting the sum of discounted risk premia from this present value. By using the risk-premium estimates derived from the historical Saudi real gross consumption per capita (and the corresponding discount rate value in Subsection 2.2) for a relative risk aversion of 2, taking into account the risk premium is here equivalent to reducing the expected oil price by 12 dollars (under the lognormal assumption) or 17 dollars (under the random walk assumption). This reduction in the expected oil price lies between 6.3 to 8.4 dollars for a relative risk aversion equal to unity, and between 17.6 and 25.3 dollars for a relative risk aversion of 3.

This risk premium is far from being negligible. Even if profitable at current oil price levels, public investment opportunities in alternative energies or energy efficiency may therefore yield lower NPVs than one might expect at first sight. Considering the risk premium may particularly impact the decision made for projects whose breakeven price is relatively close to the expected market price. Taking into account the risk premium may therefore influence the total amount of public funds that could be invested in projects aiming to curb the growth in domestic oil demand. Additionally, standard economics would generally recommend aligning domestic administered prices of power or transportation fuels with corresponding marginal costs of production or market prices. It might be noted that, as a second-order effect, the resulting decrease in domestic demand would augment the Saudi economy's exposure to oil-price volatility.

Furthermore, projects transforming oil into products less correlated with the Saudi economy generate a benefit from reducing the aggregate risk in the economy. This benefit can be priced as the present value of the corresponding negative risk premia (which represent positive cash flows). Given the above discussion about the magnitude of the risk premium, the resulting increase in project's profitability may be significant.

This paper provides estimates that could serve to formalize a rigorous economic framework for public investment decision-making in Saudi Arabia, an issue especially relevant at this time as many energy-related investment opportunities are being considered by Saudi authorities. Furthermore, the great magnitude of the risk premium derived here suggests that similar computations should be performed for other resource-rich nations whose domestic income significantly depends on the market price of an exported commodity. As far as we know, there was so far no empirical literature on this subject. The methodological approach developed in this paper, especially the estimation of the derived risk-premium formula with cointegration techniques, is a straightforward process that could be transposed to other resource-rich countries.

ACKNOWLEDGMENTS

The authors are indebted to James Smith, Christian Gollier, Bashir Dabbousi, Dermot Gately, Mustafa Babiker and three anonymous referees for helpful comments on this paper.

REFERENCES

Chetty, R. (2006). "A new method of estimating risk aversion." *The American Economic Review* 96(5): 1821–1834. http://dx.doi.org/10.1257/aer.96.5.1821.

Dasgupta, P. (2008). "Discounting climate change." *Journal of Risk and Uncertainty* 37(2): 141–169. http://dx.doi.org/10.1007/s11166-008-9049-6.

Dibooglu, S., and E. Aleisa (2004). "Oil prices, terms of trade shocks, and macroeconomic fluctuations in Saudi Arabia." *Contemporary Economic Policy* 22(1): 50–62. http://dx.doi.org/10.1093/cep/byh005.

Durand-Lasserve, O., A. Pierru and Y. Smeers (2010). "Uncertain long-run emissions targets, CO_2 price and global energy transition: a general equilibrium approach." *Energy Policy* 38(9): 5108–5122. http://dx.doi.org/10.1016/j.enpol.2010.04.041.

Gately, D., N. Al-Yousef N. and H. Al-Sheikh (2012). "The rapid growth of domestic oil consumption in Saudi Arabia and the opportunity cost of oil exports forgone." *Energy Policy* 47(August): 57–68. http://dx.doi.org/10.1016/j.enpol.2012.04.011.

Gollier *et al.* (2011). "Le calcul du risque dans les investissements publics." Report 36, Centre d'Analyse Strategique, Paris.

Gollier, C. (2012). *Pricing the planet's future: the economics of discounting in an uncertain world*. Princeton: Princeton University Press. pp. 177–179.

Gollier, C. (2007). "Comment intégrer le risque dans le calcul économique?" *Revue d'Economie Politique* 117(2): 209–223.

Gollier, C. (2001). *The economics of risk and time*. Cambridge: MIT Press.

Hall, R. and C. Jones (2007). "The value of life and the rise in health spending." *The Quarterly Journal of Economics* 122(1): 39–72. http://dx.doi.org/10.1162/qjec.122.1.39.

Kohli, U. (2004). "Real GDP, real domestic income, and terms-of-trade changes." *Journal of International Economics* 62(1): 83–106. http://dx.doi.org/10.1016/j.jinteco.2003.07.002.

Lucas, R.E. (2003). "Macroeconomic priorities." *American Economic Review* 93(1): 1–14. http://dx.doi.org/10.1257/000282803321455133.

MacDonald, R. (2010). "Real gross domestic income, relative prices, and economic performance across the OECD." Review of Income and Wealth 56(3): 498–518. http://dx.doi.org/10.1111/j.1475-4991.2010.00399.x.

Mehrara, M. and K.N. Oskoui (2006). "The sources of macroeconomic fluctuations in oil exporting countries: a comparative study." *Economic Modelling* 24(3): 365–379. http://dx.doi.org/10.1016/j.econmod.2006.08.005.

Rosser, J. B. and R.G. Sheehan (1995). "A vector autoregressive model of the Saudi Arabian economy." *Journal of Economics and Business* 47(1): 79–90. http://dx.doi.org/10.1016/0148-6195(94)00025-9.

Van Ewijk, C. and P. Tang (2003). "How to price the risk of public investment?" *De Economist* 151(3): 317–328. http://dx.doi.org/10.1023/A:1024454023591.

Weitzman, M. L. (2009). "On modeling and interpreting the economics of catastrophic climate change." *Review of Economics and Statistics* 91(1): 1–19. http://dx.doi.org/10.1162/rest.91.1.1.

APPENDIX A

Note: ***, **, and * denote statistical significance at 1%, 5%, and 10% levels, respectively; t-statistics are given in brackets.

Table A1: Unit Root Tests with Intercept (1987–2010)

		Level		*First Difference*	
	Test	Test statistic	Significance level	Test statistic	Significance level
Real GDP per capita	ADF	−2.18	—	−4.94	***
	PP	−2.18	—	−5.01	***
Command-basis GDP per capita	ADF	−1.42	—	−5.27	***
	PP	−1.37	—	−5.27	***
Log of command-basis GDP per capita	ADF	−1.35	—	−4.84	***
	PP	−1.35	—	−4.84	***
Real gross consumption per capita	ADF	−0.64	—	−3.59	**
	PP	−0.42	—	−3.59	**
Log of real gross consumption per capita	ADF	−0.71	—	−3.66	**
	PP	−0.56	—	−3.70	**
Real private consumption per capita	ADF	−2.53	—	−3.25	**
	PP	−0.36	—	−3.20	**
Real crude oil price	ADF	−1.38	—	−5.98	***
	PP	−1.28	—	−5.98	***
Log of real crude oil price	ADF	−1.35	—	−5.08	***
	PP	−1.37	—	−5.16	***

Critical values in level: −2.64 (10%); −3.00 (5%); −3.75 (1%); in first difference: −2.64 (10%); −3.00 (5%); −3.77 (1%)

Table A2: Unit Root Tests with Intercept for Percentage Change Variables (1988–2010)

Test	Real GDP per capita	Command-basis GDP per capita	Real gross consumption per capita	Real private consumption per capita	Real oil price
ADF	−4.94***	−4.72***	−3.64**	−3.17**	−5.11***
PP	−5.01***	−4.73***	−3.68**	−3.13**	−5.24***

Critical values: −2.64 (10% level); −3.00 (5% level); −3.77 (1% level)

Table A3: VEC Model of Real Gross Consumption Per Capita and Real Crude Oil Price (1987–2010)

	ΔC_t	ΔP_t
Constant	−16,697*** [−48.91]	
P_{t-1}	−222.86*** [−20.31]	
Adjustment coefficient	−0.963** [−2.86]	0.00146 [0.49]
ΔC_{t-1}	0.437** [2.17]	0.00226 [1.27]
ΔP_{t-1}	−86.94 [−1.67]	−0.23 [−0.50]
R^2	0.409	0.179
Akaike information criterion	23.002	
Schwarz information criterion	23.449	

Table A4: VEC Model of Real Private Consumption Per Capita and Real Crude Oil Price (1987–2010)

	ΔH_t	ΔP_t
P_{t-1}	−8,177*** [−10.57]	
Constant	−206.34*** [−7.65]	
Adjustment coefficient	−0.533*** [−4.26]	−0.0015 [−0.70]
ΔH_{t-1}	−0.243 [−1.15]	0.0069* [1.99]
ΔH_{t-2}	−0.691** [−2.64]	−0.0089* [−2.05]
ΔP_{t-1}	−34.23 [−1.74]	−0.454 [−1.40]
ΔP_{t-2}	−1.20 [−0.05]	−0.111 [−0.30]
R^2	0.619	0.463
Akaike information criterion	21.741	
Schwarz information criterion	22.388	

Table A5: VEC Model of Command-basis GDP Per Capita and Real Crude Oil Price (1987–2010)

	ΔI_t	ΔP_t
Cointegration constant	−20,089 [NA]	
P_{t-1}	−587.46 [NA]	
Adjustment coefficient	−0.636*** [4.26]	0
ΔI_{t-1}	1.01 [0.87]	0.00032 [0.16]
ΔI_{t-2}	0.66 [0.76]	0.0012 [0.76]
ΔP_{t-1}	−709.1 [−1.08]	−0.46 [−0.40]
ΔP_{t-2}	−484.11 [−1.00]	−0.641 [−0.75]
Error-correction constant	770.40 [0.68]	1.02 [0.51]
R^2	0.138	0.127
Akaike information criterion	23.890	
Schwarz information criterion	24.587	

Table A6: Saudi Arabian Income, Consumption, and Oil Price Data in the Period of 1987–2010

Year	World CPI (1999 base year)	Real price of a barrel of Arabian Light (1999 USD)	Real GDP per capita, at 1999 prices (SAR)	Command-basis GDP per capita (1999 SAR)	PPP converted GDI per Capita (2005 USD)	Real gross final consumption per capita (1999 SAR)	Real private final consumption per capita (1999 SAR)
1987	42.6	40.42	31,138	39,962	11,292	24,742	14,901
1988	47.5	28.23	31,710	37,035	11,982	22,832	14,443
1989	54.4	29.79	30,334	35,922	11,504	23,467	14,179
1990	56.4	36.91	31,314	41,536	12,063	23,986	14,750
1991	57.3	30.42	32,617	40,940	13,334	25,431	14,309
1992	61.1	29.36	32,386	39,845	13,805	23,886	14,275
1993	63.2	24.81	31,425	35,741	13,137	22,324	14,394
1994	73.8	20.86	30,947	32,948	12,122	21,297	14,107
1995	79.9	20.93	30,324	32,474	11,646	20,518	13,650
1996	86.3	23.06	30,543	33,992	11,557	21,329	13,691
1997	91.3	20.48	30,593	32,529	11,544	21,880	13,516
1998	97.2	12.55	30,665	27,674	10,330	20,578	12,696
1999	100.0	17.45	29,720	29,720	11,282	20,331	12,620
2000	104.1	25.76	30,437	34,833	13,982	21,987	12,896
2001	107.9	21.38	29,995	32,064	13,086	21,592	12,685
2002	110.8	21.95	29,300	31,641	13,581	21,372	12,605
2003	115.4	23.99	30,796	34,545	14,891	21,503	12,469
2004	118.9	29.04	31,640	38,669	16,527	22,639	12,819
2005	123.8	40.50	32,300	45,640	19,659	24,208	13,486
2006	129.4	47.18	32,223	48,202	21,151	25,997	14,379
2007	135.9	50.60	31,788	48,695	21,984	27,871	16,365
2008	148.1	64.25	32,050	54,467	26,703	28,185	16,387
2009	152.1	40.35	31,023	41,117	22,254	28,428	16,906
2010	159.5	48.75	31,743	45,712	—	28,568	17,137

Sources: World Bank (world CPI), *Saudi Arabian Monetary Agency* (population until 2009, nominal price of Arabian Light, real GDP, nominal exports, Saudi Arabian cost of living index), *Saudi Central Department for Statistics & Information* (2010 population, nominal and post-1997 real private final consumption and gross final consumption), *Penn World Tables* (PPP converted GDI)

APPENDIX B

The number of oil barrels that will be exported next year, denoted as q, is assumed to be known; this assumption considerably simplifies the developed expression of $var(I_1)$. By approximating that exports are all oil-related, we have:

$$I_1 = real\ \ GDP_1 + (P_1 - \overline{P})q$$

Where P_1 is the real oil price (i.e., deflated with the World CPI) in the subsequent year and $\overline{P}$ is the oil price in 1999 (i.e., the price used to compute the real GDP). By defining here nonoil GDP as real GDP minus oil exports at 1999 price, we have:

$$I_1 = nonoil\ \ GDP_1 + P_1 q$$

Hence, (7) can be rewritten:

$$\lambda = \frac{\alpha\, var(I_1)}{2E^2(I_1)} = \frac{\alpha(var(nonoil\ GDP_1) + 2q\, cov(P_1, nonoil\ GDP_1) + q^2 var(P_1))}{2E^2(I_1)}$$

As $cov(P_1, I_1) = cov(P_1, nonoil\ GDP_1) + q\, var(P_1)$, we have:

$$\lambda = \frac{\alpha\, var(nonoil\ GDP_1)}{2E^2(I_1)} + \frac{\alpha q}{2E(I_1)}\left(cov\left(P_1, \frac{I_1}{E(I_1)}\right) + cov\left(P_1, \frac{nonoil\ GDP_1}{E(I_1)}\right)\right)$$

Which gives:

$$cov\left(P_1, \frac{I_1}{E(I_1)}\right) = \frac{E(I_1)}{q}\left(\frac{2\lambda}{\alpha} - \frac{var(nonoil\ GDP_1)}{E^2(I_1)}\right) - cov\left(P_1, \frac{nonoil\ GDP_1}{E(I_1)}\right) \qquad \text{(B1)}$$

We use (B1) to calibrate the risk premium that should be considered in 2010 from a 2009 perspective and the risk premium that should be considered in 2009 from a 2008 perspective.

Let us first note that, whatever the year considered, (8) gives:

$$\frac{2\lambda}{\alpha} \cong \frac{var(G)}{(1 + E(G))^2} \cong 1.14\%.$$

In addition, it can be noticed that:

$$cov\left(P_1, \frac{nonoil\ GDP_1}{E(I_1)}\right) = \frac{E(nonoil\ GDP_1)}{E(I_1)} cov\left(P_1, \frac{nonoil\ GDP_1}{E(nonoil\ GDP_1)}\right)$$

The term $cov\left(P_1, \frac{nonoil\ GDP_1}{E(nonoil\ GDP_1)}\right)$ can be interpreted as the oil price risk premium with respect to nonoil GDP. This term, here multiplied by a factor smaller than unity, is certainly negligible since the cost of macroeconomic risks assessed with real GDP is smaller by more than one order of magnitude than that determined with the command-basis GDP.

Let us now calibrate the risk premium in 2010 from a 2009 perspective. The expected value of the command basis in 2010 is the value realized in 2009, i.e. 41,117 SAR per capita, times one plus the expected growth rate 1.16%:

$$E(I_1) = 41{,}117 \times 1.0116 = 41{,}594$$

$var(nonoil\ GDP_1)$ can be approximated as the squared value of the nonoil GDP realized in 2009 (that amounts to 25,110 SAR per capita) times the variance of real GDP growth rate. We consequently have:

$$\frac{var(nonoil\ GDP_1)}{E^2(I_1)} = \frac{0.07}{100}\left(\frac{25{,}110}{41{,}594}\right)^2 = 0.026\%$$

According to SAMA 47th annual report, Saudi Arabia exported 2,772.15 million barrels of crude oil and refined products in 2010, which represents $q = 101$ barrels of oil per capita. Therefore, (B1) gives:

$$cov\left(P_1, \frac{I_1}{E(I_1)}\right) = \frac{41594}{101}(1.14\% - 0.026\%) = 4.58$$

This figure, expressed in SAR, is computed with respect to an oil price expressed in 1999 dollars. The value of the covariance in 2009 U.S. dollars is consequently:

$$4.58 \times \frac{1.521}{3.75} = 1.86$$

As a result, with a relative risk aversion coefficient of two and from a 2009 perspective, the risk premium in 2010 would have amounted to 3.72 dollars per barrel. From the 2008 perspective and applying similar calculations, the risk premium in 2009 would have been five dollars per barrel in 2008 dollars.