# Convergence of Spot prices in the context of European Power Market Coupling

M. R. Hesamzadeh
KTH Royal Institute of Technology
mrhesamzadeh@ee.kth.se

L. Autran
KTH Royal Institute of Technology and Indar Energy
autran_lud@metz.supelec.fr

***Abstract*- In this paper, we investigate the convergence pattern of spot prices for the Central West Europe (CWE) market. We model the price series of PNX, EEX, BLX, and APX using the mean-reverting jump-diffusion geometric Brownian motion. Then we study the evolution of the parameters of this model applied to the price series of PNX, EEX, BLX, and APX in the context of CWE coupling policy. Based on our statistical analysis the convergence among the four markets is clear, but subject to shocks and it is not constant. Through our analysis we do not observe significant impacts of the recent events such as Fukushima on the convergence pattern but we can distinguish steps of convergence.**

***Index Terms—Market coupling, Jump-diffusion Mean-reverting Geometric Brownian motion***

## I. Introduction

DEREGULATION of the electricity markets has been implemented all over Europe in the past decade. The creation of regional market couplings integrating these different designs of electricity markets, is the next step toward a single and unified European market. The biggest coupling created so far is the recently extended TLC (Tri Lateral Coupling) that includes the French, German-Austrian, Dutch, and Belgian power markets in the Central West Europe, CWE, area, [1]. Such a mechanism is supposed to improve the security of supply, to optimize the cross border transmissions, and also to improve the markets liquidity. Market coupling should also result in lower price differences between the involved countries and even identical day-ahead prices.

Obviously, electricity is not a common commodity; the storage impossibility implies a very unclear relationship between spot and futures prices. Moreover, estimating long term prices by considering current spot prices is a rather complicated, if not unrealistic task, especially when one observes the growing share of renewable and consequently unpredictable production portfolios. As a result, short and long term market must be considered as two separate markets. The first one being mostly influenced by momentary variations of numerous variables such as temperature, plants production availability, demand, while the second one reflects actors' visions of tomorrow's market considering possible evolutions of energy mix, geopolitical changes, network improvements, behaviour of related commodities. In that context, it seems interesting to study the evolution of the spot price convergence in order to assess the market integration.

This paper aims at analysing the convergence phenomenon using the spot price historical data in the CWE market. We study the evolution pattern of the spot prices of the CWE market through time. This question concerns market players in order to implement hedging strategies. Steady relations between the CWE spot prices and stable convergence process allows for international hedging approaches while unstable situations can lead to local hedging strategies. It will highlight the trends that were observed before and after the coupling for spot prices in France, Germany, Belgium and Netherlands in the context of massive changes in energy policies and significant integration of renewable energy production units.

In this paper, we use the dynamic mean-reverting jump-diffusion parameters estimation approach to analysis the convergence phenomenon for the spot prices in the CWE market. In our approach we first model the CWE prices of the APX, EEX, PNX, BLX markets as the mean-reverting jump-diffusion geometric Brownian motion. Then we estimate the parameters of price models over the time. These parameters have important information regarding the behaviour of the spot prices in four coupled markets, APX, EEX, PNX, and BLX. The results of our analysis are plotted and discussed.

This paper is organised in four sections, Section I is the introduction. The mean-reverting jump-diffusion parameters estimation methodology is explained in section II. Section III applies the mean-reverting jump-diffusion parameters estimation methodology to the spot prices in four coupled markets, APX, EEX, PNX, and BLX. The results are discussed and compared. Section IV concludes the paper.

## II. Mean Reverting Jump Diffusion Parameters Estimation Approach

Modelling electricity spot prices is not an easy task. Electricity price formation is driven by supply and demand equilibrium. Inelastic demand implies the occurrence of spikes in periods of tight supply-demand balance or extreme temperatures. In addition, as most of the commodities, electricity prices tend to return to a long term mean level due to the demand and supply characteristics. The selected model for price must be able to catch these stylized features (mean reversion and jumps) in order to describe the price dynamics as accurately as possible.

A geometric Brownian motion is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion, [2], [3]. A stochastic process $S_t$ is said to follow the geometric Brownian motion if it satisfies the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \qquad (1)$$

Where $\mu$ and $\sigma$ are the drift and the volatility and $W_t$ is a standard Brownian motion.

The solution of the stochastic differential equation in (1), given the initial value $S_0$, is thus:

$$S_t = S_0 e^{(\mu - \sigma^2/2)t + \sigma W_t} \qquad (2)$$

The geometric Brownian motion is not adapted to the mean reversion features of energy commodities. Such a feature can be modelled by a mean reverting process such as:

$$dS_t = \alpha(\mu - S_t)dt + \sigma dW_t \qquad (3)$$

Where $\alpha$, $\mu$ and $\sigma$ are the strength of the mean reversion, the long-term mean level, and the volatility, respectively. The drift term (the first term on the right-hand side of equation (3)) includes the mean regression. When prices are above the long term mean level, they will tend to move downward, and the other way round.

The pure geometric Brownian motion does not model the jumps or spikes that might occur in a stochastic process such as price. A way to integrate these sudden jumps is to add a Poisson process into classical Brownian motion.

$$dS = a(S,t)dt + b(S,t)dW + \phi dq \qquad (4)$$

$q$ is a Poisson process defined by $dq = 0$ with probability $\lambda$ and $dq = 1$ with probability $1-\lambda$. $\phi$ is the size of the jump which can be a stochastic variable. The jump part can be represented by other processes but the Poisson process is the most frequent and probably the most intuitive.

After a spike, electricity prices usually tend to return to a "normal regime" and to revert to their long-term mean-value. Thus, it is logical to combine jump diffusion and mean reverting models into one model, [4], [5]. The mean-reverting jump-diffusion model is represented by the following stochastic differential equations:

$$dS_t = \alpha(\mu - S_t)dt + \sigma dW_t + J_t dP_t \qquad (5)$$

With $J_t$ the jump amplitude and $P_t$ a standard Poisson process with associated intensity $\lambda$. In this paper, we employ the *mean-reversion jump-diffusion geometric Brownian motion* to model the stochastic prices in the CWE market.

To investigate the convergence process of the prices in the CWE market, we first model the price as a mean-reverting jump-diffusion geometric Brownian motion and then we study the evolution of the price model parameters through time. To do so, we will use the maximum likelihood estimation method when an analytic formulation of the probability distribution of the model is known. For a probability distribution $D$, the associated density function $f$ and the unknown distribution parameter $\mu$, the likelihood function for a set of data from the observations $\{S_t\}_{t=1}^{N}$ is defined by: $L(\{S_t\}_{t=1}^{N}, \theta) = f(S_1 \mid \theta) f(S_2 \mid \theta) \ldots f(S_N \mid \theta)$. The likelihood function can actually be considered to have the joint density function where the observed values $\{S_t\}_{t=1}^{N}$ are fixed and $\mu$ is variable. Therefore, finding the best estimate for $\mu$ is equivalent to maximising the likelihood function. We use the log-likelihood function in our study. The best estimate is:

$$\hat{\theta} = \arg\max_{\theta}(\ln(L(\{X_t\}_{t=1}^{N}, \theta)) = \arg\max_{\theta}(\sum_{i=1}^{N} \ln(f(X_i|\theta)))$$

In order to obtain an analytical form of the characteristic function for an affine jump diffusion model leading to an analytical expression for the likelihood function, we convert the continuous formulation of the model into a discrete one by simply approximating $dt$ by $\Delta t$, [6], [7], [8], and [9]. We assume that during a small interval $\Delta t$ the probability that two or more jumps are occurring is negligible. The probability that one jump is occurring is $\lambda \Delta t$ and the probability that there is no jump is $1 - \lambda \Delta t$. The jumps are described by a Bernoulli model in the interval $\Delta t$. The jump amplitude is considered to follow a normal distribution with mean $\mu_J$ and variance $\delta$. This considerably simplifies the problem since we can now write the model as a Gaussian mixture. By approximating the continuous model with a discrete one on a small interval $\Delta t$, we obtain the density function as the product of two Gaussian density functions with and without a jump, weighted by the jump probability:

$$g(S_{t+1} \mid S_t) = \lambda \Delta t \times f_{\Delta S - \alpha(\mu - S)\Delta t + \mu_J} + (1 - \lambda \Delta t) \times f_{\Delta S - \alpha(\mu - S)\Delta t}$$

With $f_{\Delta S - \alpha(\mu - S)\Delta t + \mu_J}$ and $f_{\Delta S - \alpha(\mu - S)\Delta t}$ being the density probability functions of $\Delta S - \alpha(\mu - S)\Delta t + \mu_J$ and $\Delta S - \alpha(\mu - S)\Delta t$. The log likelihood function is then:

$$\log L(\theta \mid \{S_t\}_{t=1}^{T}) = \sum_{t=1}^{T} Ln(\lambda f_{\Delta S - \alpha(\mu - S)\Delta t + \mu_J} + (1-\lambda) f_{\Delta S - \alpha(\mu - S)\Delta t})$$

Therefore estimating the parameters is equivalent to maximizing the likelihood function.

A preliminary estimation for jump part is performed as followed:

$$R_t = Ln(\frac{S_t}{S_{t-1}}) = S_t - S_{t-1}$$

We count the number of jumps on the sample length. We consider that a jump occur if the value is higher than three times of standard deviations.

## III. APPLICATION TO THE CWE MARKET (PNX, EEX, BLX, AND APX PRICE SERIES)

Since it is not easy to estimate mean-reverting jump-diffusion geometric Brownian motion with time dependant parameters, we estimate the evolution of these parameters for four price series. Here the notion of convergence is perceived as the convergence of the model parameters. This analysis will be done in several steps. First we will estimate, for each price series, the dynamics of the five parameters of mean-reverting jump-diffusion geometric Brownian motion. For a price series, $\{S_t\}_{t=1}^{N}$ modelled by (5), we implement the maximum likelihood estimation method described above on an interval [k,k+I] with I being the fixed interval length, and k moving from 0 to N-I.

We can now proceed to the dynamic estimation of the parameters. We estimate the parameters for the jump-diffusion mean-reverting geometric Brownian motion described in (5). These parameters are: (1) $\alpha$ : mean reversion rate, (2) $\mu$ : mean reversion level, (3) $\sigma$ : stochastic diffusion volatility, (4) $\lambda$ : jump intensity, (5) $\mu_J$ : mean jump amplitude, (6) $\sigma_J$ : jump volatility. We use the MATLAB function MRJD_SIM implemented in [9] to derive these parameters for four price series of PNX, EEX, BLX, and APX in CWE market.

The interval length of the estimation is chosen to be I=250 as it corresponds approximately to working days of one year. Since we have 1250 days (5 years), k=1000 and this is equivalent to 1000 estimations for the parameters from the interval $I_0$ =[1,250] up to $I_5$ =[1000,1250]. The evolution of the parameters of the mean reverting jump diffusion geometric Brownian motion model for price series of PNX, EEX, BLX, and APX are illustrated in Fig. 1, Fig. 2, Fig. 3, and Fig. 4, respectively.

Apart from some small disturbances, the four price series show the same dynamics for each parameter. We can notice that there is a big discontinuity for the mean reversion rate, the volatility and the mean jump amplitude of PNX around *k=500* which correspond to the exceptionally high price level reached in 2009 on the French power exchange.

We also observe that the volatility, the jump intensity, and standard jump deviation seem to decrease through the time for each market which could be a sign of better integration. It is however harder to find a common pattern for the mean jump amplitude which is the parameter that reflects the erratic behaviour of prices.

The shape of the mean reversion levels are very similar from k = 0 to k = 250, they are increasing, which is logical because it corresponds to the rise observed in energy prices between 2006 and 2008.

Then they decrease and reach their lowest level for k = 500, which corresponds to the end of 2008 and the beginning of 2009 so that these parameters are estimated for the year 2009, when the economic crisis happened.

To compare these parameters, the evolution of the difference of these parameters are calculated and plotted. First we compute the difference between the PNX and EEX parameters of the mean-reverting jump-diffusion geometric Brownian motion model. This is illustrated in Fig. 5.

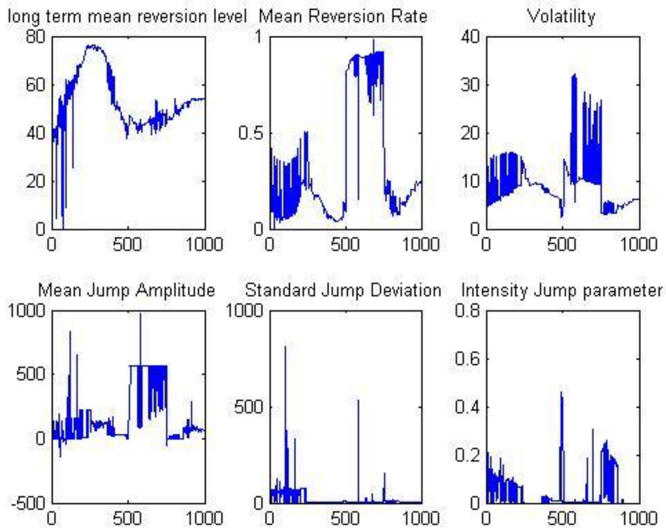


Fig. 1 Evolution of parameters of the mean reverting jump diffusion geometric Brownian motion model for PNX price series

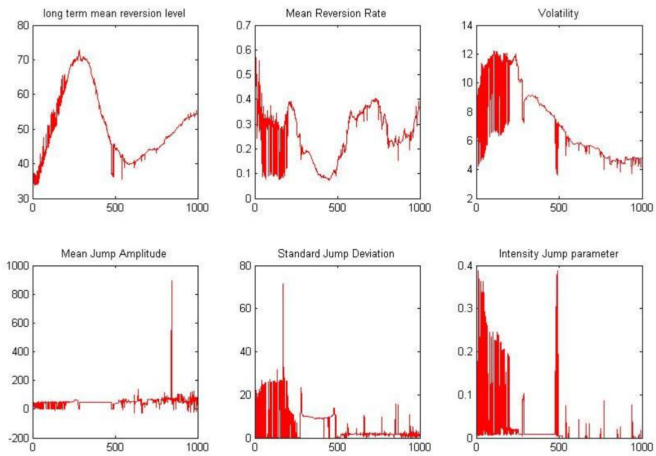


Fig. 2 Evolution of parameters of the mean reverting jump diffusion geometric Brownian motion model for EEX price series

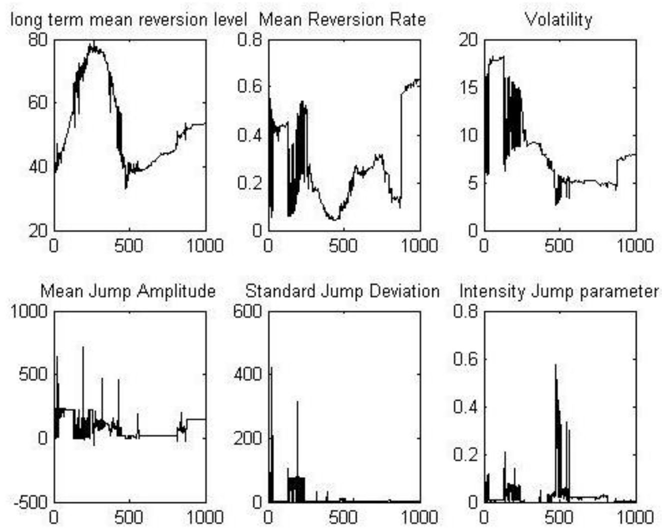


Fig. 3 Evolution of parameters of the mean reverting jump diffusion geometric Brownian motion model for BLX price series

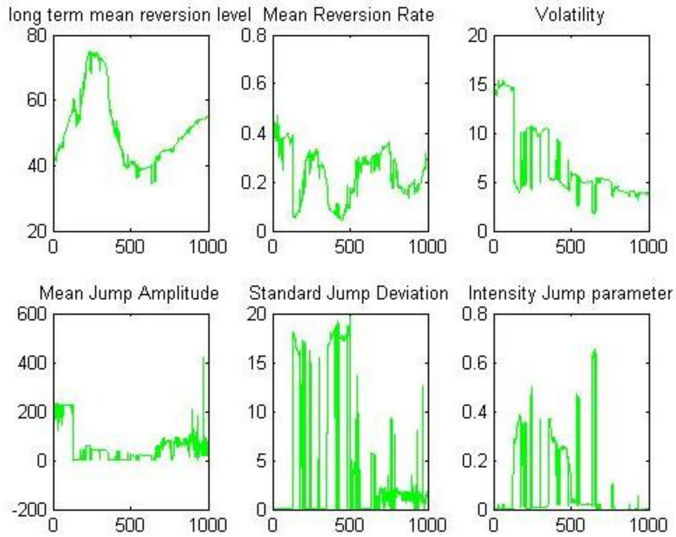


Fig. 4 Evolution of parameters of the mean reverting jump diffusion geometric Brownian motion model for APX price series

The most relevant parameter in our study is the mean reversion level because it symbolises the spot electricity price during normal condition by taking apart the stochastic shocks of supply and demand that are included in the jump parameters.

We observe that this difference is globally decreasing and converging toward zero, as highlighted by the red line which is a quadratic fitting. The EEX and PNX are converging toward common mean reversion levels. However, when looking closer at the curve we notice that such convergence is "stepwise". These different steps are indicated in Fig. 6. For the other parameters, it is harder to perceive a clear pattern but we observe that their differences, apart from the mean reversion rate, generally tend to stabilize around 0 which is obviously a sign of convergence (given we neglect the sudden but temporary increase in the mean reversion rate, jump amplitude and volatility due to the price spike of PNX in 2009).

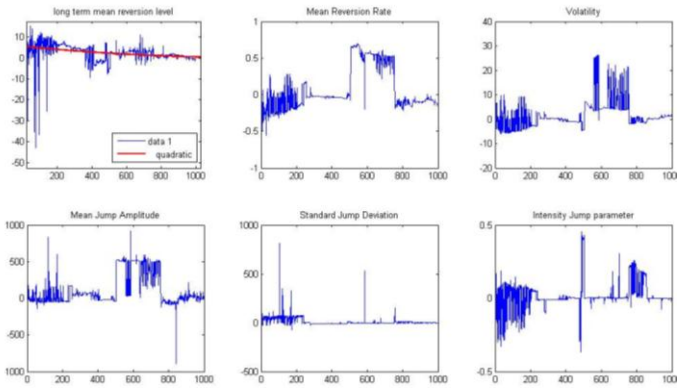


Fig. 5 Difference of parameters between PNX and EEX

In order to assess the convergence process for the four price series simultaneously, the maximum difference of the parameters is plotted in Fig. 7.

From this point of view, it is rather clear that the four markets are converging since, for most of the parameters, the maximum difference is heading towards zero. The standard deviation of jumps and to some extent the volatility also seems to converge. For three remaining parameters, it is less clear because: either they are impacted by the big price spikes that disturb estimation (especially the mean reversion rate for k around 500, 750 and 900), or they represent the jump part. These jumps can occur in one country independently from the others.

The statistical analysis shows the convergence among the four markets, but convergence is subject to shocks and it is not constant. Through this method we do not observe significant impacts of the recent events such as Fukushima but we can distinguish steps of convergence.

Finally, using our model, we simulate the spot prices for the next 250 days to the end of August 2012. We compute 10000 simulations of PNX, EEX, APX and BLX with the last historical data and used them as starting points. We calculate for each four markets, 10000 simulations of prices between 7$^{th}$ September 2011 and 22$^{nd}$ August 2012 (corresponding to 250 week days).

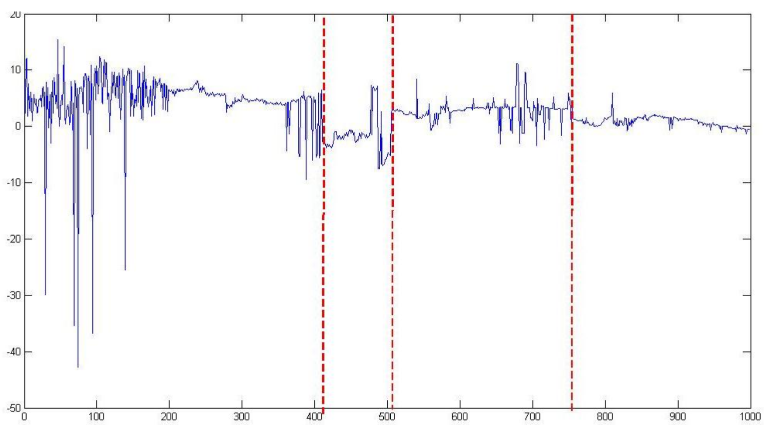

Fig. 6 Difference between mean reversion levels of PNX and EEX

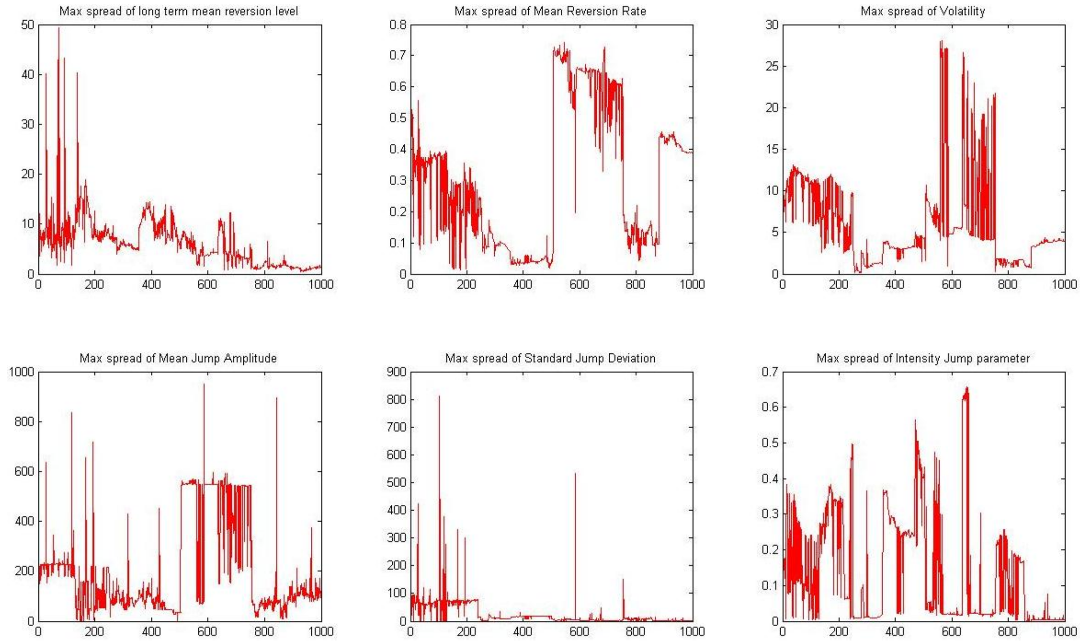


Fig. 7 Maximum difference between each parameter of the Mean Reverting Jump Diffusion geometric Brownian motion model

The estimations for the mean values of prices are collected in Table I.

TABLE I
MEAN VALUES FOR PRICES ESTIMATED THROUGH SIMULATION (PERIOD FROM $7^{TH}$ SEPTEMBER 2011 TO $22^{ND}$ AUGUST 2012 )

| | |
|---|---|
| **APX** | 54,88 €/MWh |
| **EEX** | 54,4 €/MWh |
| **PNX** | 53,81 €/MWh |
| **BLX** | 54,42 €/MWh |

As we can see the mean value of PNX remains below mean values of APX, EEX, and BLX as observed recently. The maximum difference is equal to 1.07€/MWh and the difference between EEX and PNX is equal to 0.59€/MWh.

## IV. Conclusion

This study was dedicated to explore the convergence process for the Central West Europe (CWE) market. We use the Mean-Reverting Jump-Diffusion model and estimated its related parameters (mean reversion level, mean reversion rate, volatility, jump intensity, jump amplitude, jump volatility) on a constant length interval along the historical price series. Prices clearly show signs of convergence especially through the mean reversion level. Computing the maximum and minimum differences between each parameter, we observed that the difference in mean reversion levels is decreasing, while standard deviation and jump diffusion are converging. It was however harder to observe significant moves in the other parameters.

The analysis shows that there is convergence among the CWE countries. The relation between prices is getting steadier. The dynamics of the convergence process is stepwise: jumping from one convergence state to another although impacts of external events are not clearly identified in our analysis.

However, more detailed studies are needed to explore the exact pattern of convergence in the CWE market.

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