Sectoral Interfuel Substitution in Canada: An Application of NQ Flexible Functional Forms

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ABSTRACT

This paper focuses on the aggregate demand for electricity, natural gas, and light fuel oil in Canada as a whole and six of its provinces—Quebec, Ontario, Manitoba, Saskatchewan, Alberta, and British Columbia—in the residential, commercial, and industrial sectors. We employ the locally flexible normalized quadratic (NQ) expenditure function (in the case of the residential sector) and the NQ cost function (in the case of the commercial and industrial sectors), treat the curvature property as a maintained hypothesis, and provide evidence consistent with neoclassical microeconomic theory. We find that the Morishima interfuel elasticities of substitution are in general positive and statistically significant. Our results indicate limited substitutability between electricity and natural gas, but strong substitutability between light fuel oil and each of electricity and natural gas in most cases.

Keywords: Flexible functional forms, NQ expenditure function, NQ cost function, Global concavity

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1. INTRODUCTION

Over the years, there have been two major approaches to the investigation of interfuel substitution (energy elasticities) and the demand for energy. One approach uses cointegration techniques and error-correction models to estimate long-run and short-run demand elasticities, respectively. Although this approach deals with econometric regularity issues, it lacks proper microeconomic foundations—see, for example, Bentzen and Engsted (1993) and Hunt and Manning (1989). The other approach allows the estimation in a systems context assuming a flexible functional form for the aggregator function, based on the dual approach to demand system generation developed by Diewert (1974). Using recent methodological advances in microeconometrics, this approach allows us to achieve theoretical regularity (in terms of curvature, positivity, and monotonicity of neoclassical microeconomic theory). It is difficult, however, to simultaneously achieve econometric regularity (in terms of stationary equation errors), because the combination of nonstationary data and nonlinear estimation in large demand systems is an extremely difficult issue and has not yet been addressed in the literature.

The flexible functional forms approach was pioneered by Berndt and Wood (1975), Fuss (1977), and Pindyck (1979) in the context of interfactor and interfuel substitution. It involves the specification of a differentiable form for the cost function, the application of Shephard's (1953)

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lemma to derive the cost share equations, and the use of relevant data to estimate the parameters and compute the relevant elasticity measures like the income elasticities, the own- and cross-price elasticities, and the Allen and Morishima elasticities of substitution. However, the major contributions in this area are quite outdated by now, since their data incorporate observations before the 1970s. Moreover, most of these studies ignore the theoretical regularity conditions of neoclassical microeconomic theory or do not report the results of full regularity checks. An exception is a series of recent papers by Serletis et al. (2010, 2011) and Chang and Serletis (2014) which pay explicit attention to the theoretical regularity conditions and produce meaningful inference consistent with the theory.

In this paper we take the flexible functional forms approach to examine interfuel substitution possibilities in energy demand within the residential, commercial, and industrial sectors in Canada as a whole and six of its provinces—Quebec, Ontario, Manitoba, Saskatchewan, Alberta, and British Columbia—as data limitations make it impossible to deal with all provinces. We focus on electricity, natural gas, and light fuel oil, ignoring energy forms with low shares in total energy expenditure like heavy fuel oil, kerosene, wood, and liquefied natural gas. Our objective is to provide updated empirical work using methodological improvements of the past twenty years, and improve our understanding of how changing energy prices and incomes will influence interfuel substitution and the demand for energy in the future.

In order to achieve this, we use recent state-of-the art advances in microeconometrics. In particular, we use duality theory and the demand systems approach based on neoclassical consumer and firm theories which allows us to estimate the demand for electricity, natural gas, and light fuel oil in a systems framework. We also use a locally flexible demand system derived from the expenditure/cost function of the representative consumer/firm. Moreover, we are motivated by the widespread practice of ignoring the theoretical regularity conditions of neoclassical microeconomic theory and approximate the unknown underlying expenditure function using a flexible functional form that allows the imposition of global curvature without losing its flexibility property. In particular, in the case of the commercial and industrial sectors we use the normalized quadratic (NQ) cost function, introduced by Diewert and Wales (1987), and in the case of the residential sector energy demand we use the NQ expenditure function, introduced by Diewert and Wales (1988). The NQ cost function has also been used recently by Serletis et al. (2010, 2011), but the NQ expenditure function is (to our knowledge) used for the first time in the empirical energy demand literature. It is to be noted that McKitrick (1998) also used NQ functional forms to estimate consumer demand and producer input demand in the context of computable general equilibrium models in Canada.

The rest of the paper is organized as follows. Section 2 sketches out related neoclassical theory and applied consumption and production analyses. Section 3 presents the NQ expenditure and cost functions and derives the associated systems of consumer demand and input demand functions. Section 4 discusses related econometric issues, paying explicit attention to the singularity problem and the imposition of global concavity. Section 5 discusses the data and Section 6 presents the empirical results for each of the residential, commercial, and industrial sectors. Section 7 compares the reported results to those obtained in analyses performed by others, and the final section concludes the paper.

2. THE STRUCTURE OF PRODUCTION AND PREFERENCES

Our econometric approach requires certain assumptions about the structure of production and preferences. We assume that the group of n energy inputs in the production context (or goods in the consumer context) is homothetically weakly separable from the non-energy forms in the underlying aggregator function f (production function in producer theory or utility function in utility theory). Therefore, the aggregator function f has the form

$$Q = f(E(\boldsymbol{x}),M) \tag{1}$$

where Q is gross output (or utility, u), $E(\cdot)$ is a homothetic aggregator function over the n energy inputs (or goods), $\boldsymbol{x} = (x_1,\ldots,x_n)$, and $\boldsymbol{M}$ is a vector of non-energy inputs (or goods). The requirement of weak separability in $\boldsymbol{x}$ is that the marginal rate of substitution between any two components of $\boldsymbol{x}$ does not depend upon the value of $\boldsymbol{M}$.

Under these assumptions and duality theorems [see Diewert (1974)], the corresponding cost function (or expenditure function in utility theory) can be written as

$$C = g(P_E(\boldsymbol{p}),\boldsymbol{p}_M,Q)$$

where $\boldsymbol{p} = (p_1,\ldots,p_n)$ is the corresponding price vector of the n forms of energy, $\boldsymbol{p}_M$ that of non-energy forms, and $P_E(.)$ is an energy price aggregator function which is a homothetic function and can represented by a unit cost or expenditure function.

Our objective is to estimate a system of demand equations for the residential sector and a system of input demands for each of the commercial and industrial sectors and produce inference consistent with neoclassical microeconomic theory. In order to do so, we use flexible functional forms capable of approximating an arbitrary twice continuously differentiable function to the second order at an arbitrary point in the domain. Moreover, the flexible functional forms that we use allow for the imposition of global curvature without losing their flexibility property.

3. NORMALIZED QUADRATIC FUNCTIONAL FORMS

We use the NQ expenditure function, developed by Diewert and Wales (1988), to investigate interfuel substitution possibilities in energy demand within the residential sector and the NQ cost function, developed by Diewert and Wales (1987), to investigate interfuel substitution possibilities in energy demand within the commercial and industrial sectors. In what follows, we briefly derive the demand system for the NQ expenditure function and the input demand equations for the NQ cost function.

3.1 The NQ Expenditure Function

For a given utility level and vector of prices $\boldsymbol{p}$, the NQ expenditure function is defined as

$$C(\boldsymbol{p},u) = \sum_{i=1}^{n} \theta_i p_i + u\left(\sum_{i=1}^{n} b_i p_i\right) + \frac{1}{2}u\frac{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} p_i p_j\right)}{\left(\sum_{i=1}^{n} \alpha_i p_i\right)} \tag{2}$$

where $\boldsymbol{\theta} = [\theta_1,\theta_2,\ldots,\theta_n]$, $\boldsymbol{b} = [b_1,b_2,\ldots,b_n]$, and the elements of the $n \times n$ matrix $\boldsymbol{B} \equiv [\beta_{ij}]$ are the unknown parameters to be estimated.

The elements of the non-negative vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_n]$ are predetermined. In fact, according to Diewert and Fox (2009), the $\boldsymbol{\alpha}$ vector can be either a vector of ones ($\boldsymbol{\alpha} = \mathbf{1}_n$) or the sample mean of the observed commodity vector, $\alpha = (1/T)\sum_{t=1}^{T} \boldsymbol{x}_t$. In this paper we use the former.

To ensure the flexibility and Gorman polar form of the NQ form, we follow Diewert and Wales (1988) and impose the following restrictions

$$\sum_{i=1}^{n} \alpha_i p_i^* = 1, \quad \alpha_i \geq 0 \quad \forall i \tag{3}$$

$$\sum_{i=1}^{n} \theta_i p_i^* = 0 \tag{4}$$

and

$$\sum_{i=1}^{n} \beta_{ij} p_j^* = 0 \quad \forall i \quad \text{and} \quad \beta_{ij} = \beta_{ji}, \quad \forall i,j \tag{5}$$

where $\boldsymbol{p}^* \gg \mathbf{0}_n$ is a reference (or base-period) vector of normalized prices, determined in such a way that $\boldsymbol{p}^* = \mathbf{1}_n$.

The NQ demand system in budget share form is

$$s_i(\boldsymbol{v}) = \theta_i v_i + \frac{b_i + \dfrac{\left(\sum_{j=1}^{n} \beta_{ij} v_i\right)}{\left(\sum_{i=1}^{n} \alpha_i v_i\right)} - \dfrac{1}{2} \dfrac{\left(\alpha_i \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{kj} v_k v_j\right)}{\left(\sum_{i=1}^{n} \alpha_i v_i\right)^2}}{\sum_{i=1}^{n} b_i v_i + \dfrac{1}{2} \dfrac{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} v_i v_j\right)}{\left(\sum_{i=1}^{n} \alpha_i v_i\right)}} \times \left(1 - \sum_{i=1}^{n} \theta_i v_i\right) v_i \tag{6}$$

where $\boldsymbol{v} = [v_1, v_2, \ldots, v_n]$ is the vector of income normalized prices, with the jth element $v_j = p_j/y$, and $s_i = v_i x_i$ is the share of the ith good in the total expenditure.

We can use different elasticity measures, calculated from the Marshallian demand functions, $x_i(\boldsymbol{v})$, $i = 1, \ldots, n$, to conduct empirical demand analysis—for more details, see Barnett and Serletis (2008). In particular, the own- and cross-price elasticities, η_{ij}, can be calculated as

$$\eta_{ij} = \frac{\partial x_i}{\partial v_j} \frac{v_j}{x_i}, \quad i,j = 1, \ldots, n. \tag{7}$$

We can also use the homogeneity of degree zero in $(\boldsymbol{p}, y)$ property of the Marshallian demand functions and calculate the expenditure (income) elasticities as

$$\eta_{iy} = -\sum_{j=1}^{n}, \quad i = 1, \ldots, n. \tag{8}$$

In addition, we can use the Allen-Uzawa and Morishima elasticities of substitutions to investigate substitutability/complementarity relationships among goods. In particular, the Allen-Uzawa elasticity of substitution, σ_{ij}^{a}, can be calculated as

$$\sigma_{ij}^{a} = \eta_{iy} + \frac{\eta_{ij}}{v_j x_j}. \tag{9}$$

It should be noted, however, that with more than two goods (as in our case), the Allen-Uzawa elasticity of substitution may be uninformative—see Blackorby and Russell (1989). In particular, for two goods the relationship is that of substitutability. When there are more than two goods, the relationship becomes complex and depends on things such as the direction taken toward the point of approximation. In that case the Morishima elasticity of substitution, σ_{ij}^{m}, is the correct measure of substitution

$$\sigma_{ij}^{m} = v_i x_i (\sigma_{ji}^{a} - \sigma_{ii}^{a}). \tag{10}$$

Note that the Morishima elasticity of substitution looks at the impact on the ratio of two goods, x_i/x_j. Goods will be Morishima complements (substitutes) if an increase in the price of j, p_j, causes x_i/x_j to decrease, $\sigma_{ij}^{m} < 0$ (increase, $\sigma_{ij}^{m} > 0$).

3.2 The NQ Cost Function

The cost version of equation (2) is much the same as the expenditure except that the corresponding cost function is $C(\boldsymbol{p}, y)$ in the production context, where y is gross sector output and $\boldsymbol{p}$ is the vector of input prices. The NQ cost function, developed by Diewert and Wales (1987), is given by

$$C(\boldsymbol{p},y) = y\left[\sum_{i=1}^{n} b_i p_i + \frac{1}{2}\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}p_i p_j}{\sum_{i=1}^{N}\alpha_i p_i}\right]. \tag{11}$$

We impose the same restrictions (3) and (5) to ensure the flexibility of the NQ cost function (11). Next we apply Shephard's lemma to obtain the following input demand equation for each input i in the commercial and industrial sectors

$$\frac{x_i}{y} = b_i + \sum_{j=1}^{n}\beta_{ij}\frac{p_i}{\sum_{i=1}^{n}\alpha_i p_i} - \frac{1}{2}\alpha_i\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\frac{p_i}{\sum_{i=1}^{n}\alpha_i p_i}\frac{p_j}{\sum_{j=1}^{n}\alpha_j p_j}\right). \tag{12}$$

As with the NQ expenditure function, we use equation (7) to calculate the own- and cross-price elasticities, η_{ij}, equation (9) to calculate the Allen elasticities of substitution, σ_{ij}^{a}, and equation (10) to calculate the Morishima elasticities of substitutions, σ_{ij}^{m}. In doing so, we use input prices, $\boldsymbol{p}$, instead of income normalized prices, $\boldsymbol{v}$, and sector output to measure y.

4. ECONOMETRIC CONSIDERATIONS

To estimate the share equations (6) for the residential sector and the input demands (12) for each of the commercial and industrial sectors, we add a stochastic component and write each of equations (6) and (12) as follows

$$z_t = \boldsymbol{\psi}(\boldsymbol{q}_t, \boldsymbol{\theta}) + \boldsymbol{\varepsilon}_t \tag{13}$$

where $z_t = (z_1, \ldots, z_n)'$ is the vector of expenditure shares (s_i) in equation (6) or output normalized quantities (x_i/y) in equation (12) and $\boldsymbol{q}$ denotes the corresponding price vector. $\boldsymbol{\varepsilon}_t$ is a vector of stochastic errors, and we assume that $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{\Omega})$ where $\mathbf{0}$ is a null matrix and $\boldsymbol{\Omega}$ is the $n \times n$ symmetric positive definite error covariance matrix. $\boldsymbol{\psi}(\boldsymbol{q}_t, \boldsymbol{\theta}) = (\psi_1(\boldsymbol{q}_t, \boldsymbol{\theta}), \ldots, \psi_n(\boldsymbol{q}_t, \boldsymbol{\theta}))'$, and $\psi_i(\boldsymbol{q}_t, \boldsymbol{\theta})$ is given by the right-hand side of each of (6) and (12). Notice also that we estimate only $n-1$ share equations in (6), because of the singularity problem (that is, the shares sum to 1), but we estimate n input demand equations in (12).

To impose restrictions (3) empirically, we set the prices to unity in the reference (base) year. Thus, equation (5) becomes

$$\sum_{j=1}^{n} \beta_{ij} = 0, \quad \forall i \quad \text{and} \quad \beta_{ij} = \beta_{ji}, \quad \forall i,j. \tag{14}$$

We also set $\boldsymbol{\alpha} = \mathbf{1}_n$ so that the restriction (3) reduces to

$$\alpha_i = 1, \quad \forall i.$$

In the case of the share equations (6) we also set the reference-year prices to unity, so that restriction (4) implies

$$\sum_{i=1}^{m} \theta_i = 0.$$

Finally, the demand equations, and consequently share equations (6), are homogeneous of degree zero, and we follow Diewert and Wales (1988) and impose the following restriction

$$\sum_{i=1}^{n} b_i = 1.$$

Under these restrictions, the NQ expenditure and cost systems are well defined systems. The NQ expenditure system has $(n^2 + 3n - 4)/2$ free parameters (that is, parameters estimated directly) and the NQ cost system has $(n^2 + n)/2$ free parameters.

Diewert and Wales (1987) argue that the concavity of the NQ expenditure and cost functions may not be satisfied, in the sense that the estimated B matrix may not be negative semidefinite. We follow Diewert and Wales (1988), and impose global concavity by setting $B = -KK'$, where $K = [k_{ij}]$ is a lower triangular matrix. As an example, in the case with $n = 3$ (which is the case in the empirical part of our paper), concavity of the NQ expenditure and cost functions (2) and (11) can be imposed by replacing the elements of B in (6) and (12) by the elements of K, as follows

$$\beta_{11} = -k_{11}^2$$
$$\beta_{12} = -k_{11}k_{12}$$
$$\beta_{22} = -(k_{12}^2 + k_{22}^2).$$

We can recover the other elements of B using restriction (14) as follows

$$\beta_{13} = -(\beta_{11} + \beta_{12})$$
$$\beta_{23} = -(\beta_{12} + \beta_{22})$$
$$\beta_{33} = \beta_{11} + 2\beta_{12} + \beta_{22}.$$

We also attempted to achieve econometric regularity by correcting for serial correlation by allowing the possibility of a first-order autoregressive process in the error terms of equation (13), as in Serletis et al. (2010) in the context of their NQ cost function. However, we observed that serial correlation correction increases the number of curvature violations and also leads to induced violations of monotonicity and positivity. In this regard, it should be noted that allowing for first order serial correlation, as is usually done in the literature, is almost the same as taking first differences of the data if the autocorrelation coefficient is close to unity. In that case, the equation errors become stationary, but there is no theory for the models in first differences. Moreover, even if the errors are stationary and the estimates are super consistent, as argued by Attfield (1997) and Ng (1995), standard estimation procedures are inadequate for obtaining correctly estimated standard errors for coefficients in cointegrating equations. Thus, to simultaneously achieve both economic and econometric regularity in nonlinear demand systems like the ones used in this paper seems to be a challenging task and an area for potentially productive future research.

5. DATA

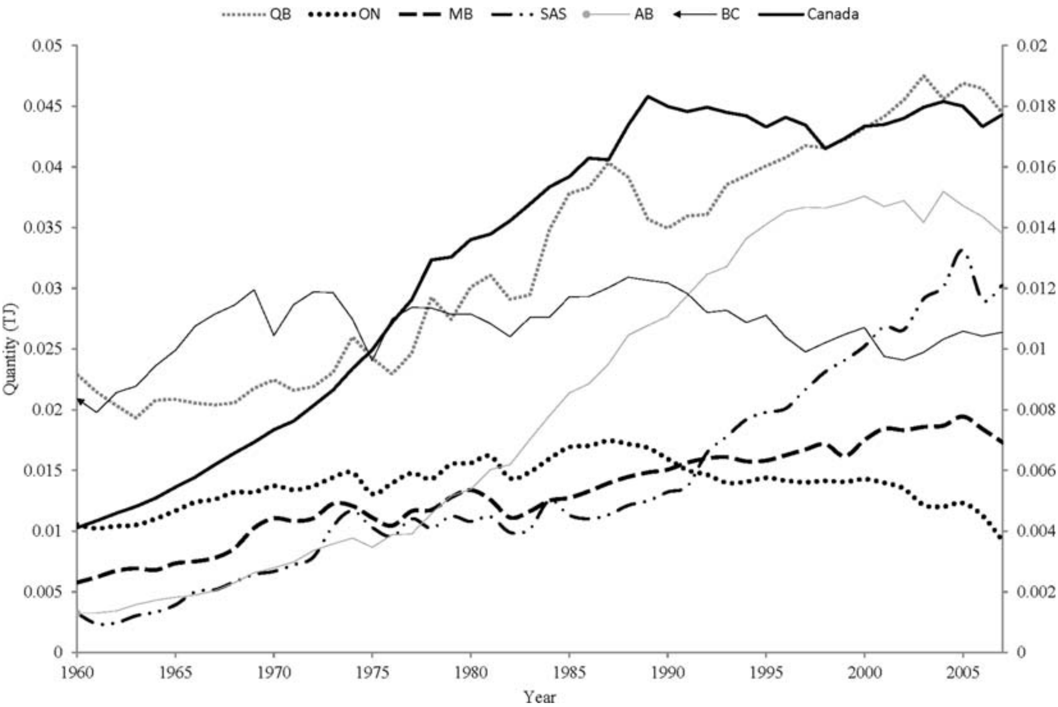
We use annual data from 1960 to 2007, a total of 48 observations. Our data is assembled from different sources, including CANSIM II and annual publications from Electric Power Statistics, Gas Utilities, and Detailed Energy Supply and Demand. The data set consists of prices and quantities of the various energy goods used in the residential, commercial, and industrial sectors in Canada as a whole, and six provinces—Quebec, Ontario, Manitoba, Saskatchewan, Alberta, and British Columbia.

For our empirical work we employ the three main sources of energy in the three sectors—electricity (e), natural gas (g), and light fuel oil (f) measured in Tera-joules (TJ). Light fuel oil (also called heating oil) includes all distillate type fuels for heating, furnace fuel oil, gas oils, and light industrial fuel. Since the energy goods are measured in different units, we convert all energy quantities in TJ, as follows

Electricity:	One Giga Watt per hour = 3.6 TJ
Natural gas:	One million cubic metres = 37.78 TJ
Light fuel oil:	1,000 cubic metres = 38.68 TJ.

We ignore other energy goods such as heavy fuel oil, kerosene, and wood, because of their small share in total energy consumption. For example, in the residential sector of Canada as a whole, the average shares (over the sample period) of heavy fuel oil and kerosene are 0.4% and 3.5%, respectively, compared to those of electricity, natural gas, and light fuel oil of 54%, 21%, and 20%, respectively. In doing so, in some cases we ignore forms of energy that are likely to be a substitute to oil, natural gas, and electricity, such as, for example, wood in the province of Quebec that accounts for up to 10% of energy consumption.

Figure 1: Per Capita Electricity Consumption: Residential Sectors



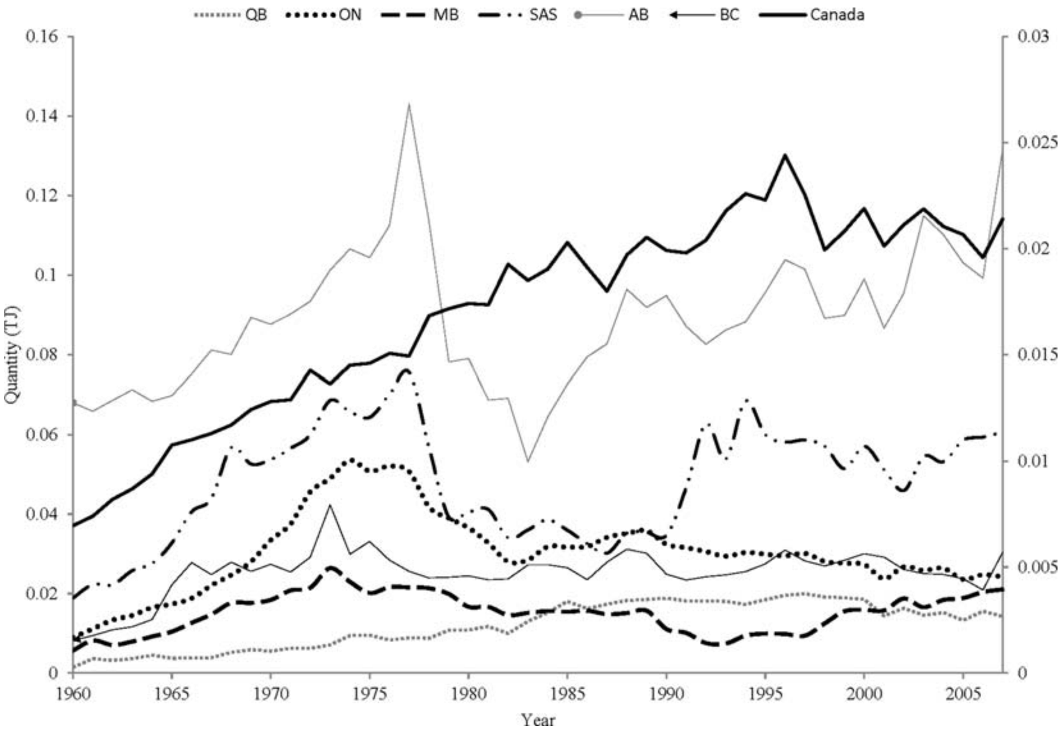
Note. Annual data, 1960–2007. Provincial consumptions are displayed in the y_1 axis and Canadian consumption in the y_2 axis.

Because it is difficult for utilities companies to separate residential and agricultural (farms) customers, we use combined residential and agricultural consumption data in our analysis of the residential sector. We also use per capita data for the residential sector, but aggregate data for the other two sectors, namely the commercial and industrial sectors. Moreover, we use national and provincial GDP for each of the industrial and commercial sectors [i.e. the variable y in equation (12)].

For each energy good (electricity, natural gas, and light fuel oil) we compute average prices by dividing energy sales revenue by quantities sold, and apply related provincial and federal tax rates to the resulting values. For Canada as a whole, the price of each good is calculated by taking the average good price in each province weighed by the quantity of the good in energy units of that province.

To understand the evolution of consumption and nominal prices of the three energy goods (electricity, natural gas, and light fuel oil) over the sample period in the residential sector, we show per capita (electricity, natural gas, and light fuel oil) consumption in Canada and the six provinces in Figures 1, 2, and 3 (with per capita consumption in Canada being displayed in the second y axis) and the corresponding prices in Figures 4, 5, and 6. As can be seen, consumption of both electricity and natural gas in the combined residential and agricultural sectors has been increasing relatively steadily over the sample period, with natural gas consumption experiencing larger fluctuations in some provinces like Alberta and Saskatchewan since 1976. Consumption of light fuel oil increased until the early 1970s, but decreased quite rapidly after that in Canada as a whole and also the six provinces. Overall, per capita energy consumption in most provinces follows the Canadian trend,

Figure 2: Per Capita Natural Gas Consumption: Residential Sectors



Note. Annual data, 1960–2007. Provincial consumptions are displayed in the y_1 axis and Canadian consumption in the y_2 axis.

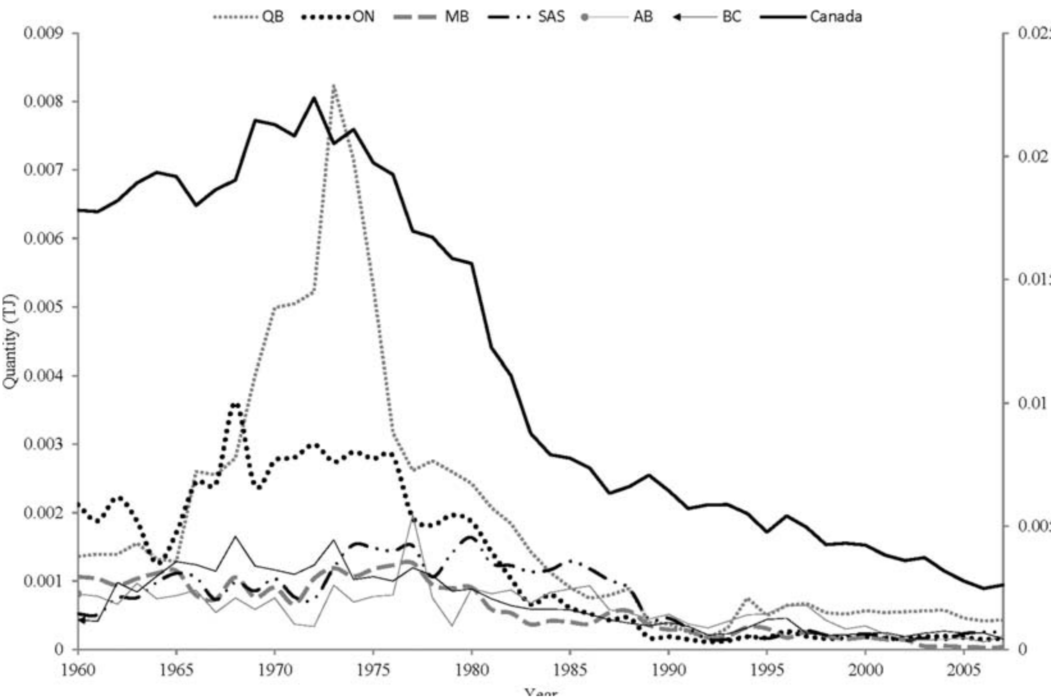
which shows an increase in electricity and natural gas demand over the sample period and a decline in that for light fuel oil. The working paper version of this article provides similar information for the commercial and industrial sectors.

6. EMPIRICAL EVIDENCE

The estimation is performed in TSP/OxMetrics (version 5.1) using the FIML procedure. We check the theoretical regularity conditions of positivity, monotonicity, and concavity as in Feng and Serletis (2008) and Serletis et al. (2010, 2011). In particular, positivity is checked by verifying that the estimated shares/input demands are positive, and monotonicity is checked by direct computation of the values of the first gradient vector of the estimated expenditure/cost function with respect to prices. Concavity is checked by examining whether the Slutsky/Hessian matrix (derived from the expenditure/cost function) is negative semidefinite. It is satisfied if the eigenvalues of that matrix are non-positive.

For the purpose of our analysis, we do not report parameter estimates, the own- and cross-price elasticities, and the Allen cross-price elasticities of substitution in this article; we report these in an Appendix in the working paper version of the article. In particular, for Canada as a whole and each of the six provinces (Quebec, Ontario, Manitoba, Saskatchewan, Alberta, and British Columbia) and for each sector (residential, commercial, and industrial), we present the results in Appendix Tables A1–A21 of the working paper in terms of parameter estimates and positivity,

Figure 3: Per Capita Light Fuel Oil Consumption: Residential Sectors



Note. Annual data, 1960–2007. Provincial consumptions are displayed in the y_1 axis and Canadian consumption in the y_2 axis.

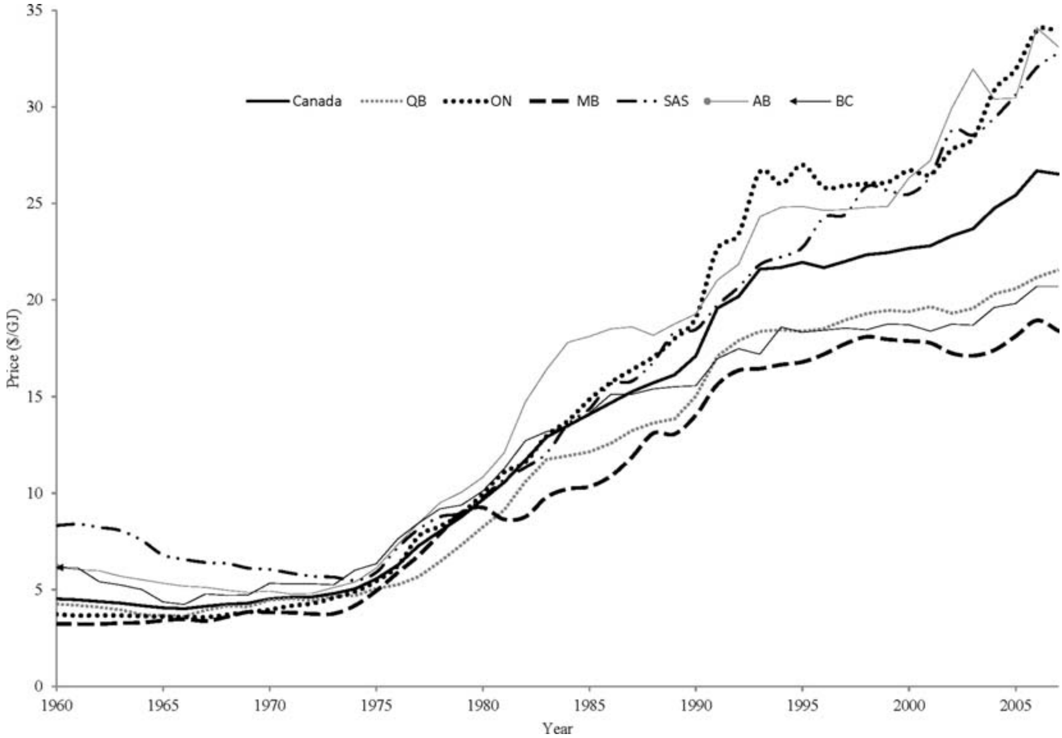
monotonicity, and concavity violations when the NQ model is estimated without the concavity conditions imposed (in the first column) and with the concavity conditions imposed when necessary (in the second column). We present the estimates of the own- and cross-price elasticities, η_{ij}, in Appendix Tables A22, A24, and A26 of the working paper, and the estimates of the Allen elasticities of substitution, σ^a_{ij}, in Appendix Tables A23, A25, and A27 of the working paper. In Tables 1–3 that follow, we only report the Allen own-price elasticities, σ^a_{ii}, and the (asymmetrical) Morishima elasticities of substitution, σ^m_{ij}, together with their *p*-values.

In all tables, '*e*' stands for electricity, '*g*' for natural gas, and '*f*' for light fuel oil, and numbers in parentheses are *p*-values. All elasticities are calculated at the mean of the data and the *p*-values have been computed by linearizing the elasticity formulas around the estimated parameter values and then by using the standard formulas for the variance of linear functions of random variables. Reported results are based on equation (13) with the curvature conditions imposed when curvature was violated in the unconstrained version of the model.

In general, although positivity and monotonicity are satisfied at many sample observations, concavity is violated at all sample observations when the concavity conditions are not imposed (see the first column of Appendix Tables A1–A21 in the working paper). In fact, only in 6 out of 21 cases the concavity condition is satisfied by luck, those being the residential sector in Quebec, the commercial sectors in Canada, Manitoba, and British Columbia, and the industrial sector in Canada and Manitoba. This is consistent with the evidence in Feng and Serletis (2008) and Serletis et al. (2010, 2011).

Because regularity has not been attained, except for the six mentioned cases, we follow the suggestions of Barnett (2002) and, as in Feng and Serletis (2008) and Serletis et al. (2010,

Figure 4: Average Electricity Prices: Residential Sectors

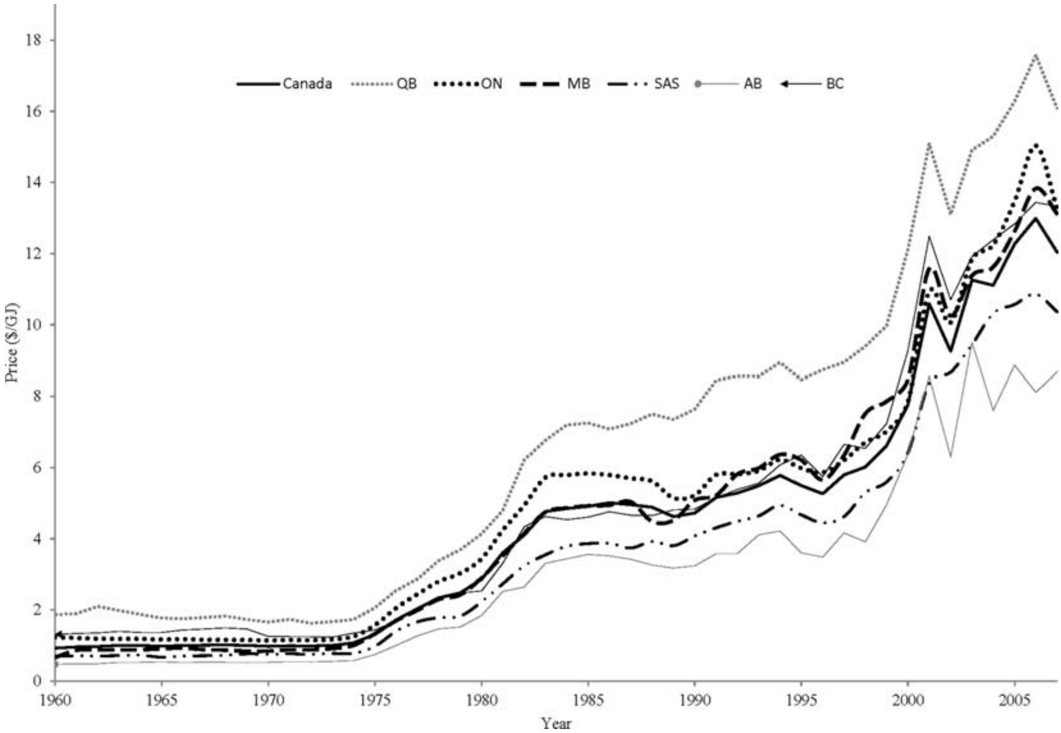


2011), we estimate the NQ expenditure model by imposing concavity following the procedure discussed in Section 4. The results, reported in the second column of Appendix Tables A1–A21 of the working paper version, (if applicable) are impressive. They indicate that imposing global concavity (at all possible prices) reduces the number of concavity violations to zero, without any induced violations of monotonicity; only in the case of the residential sector of Ontario the imposition of concavity does not ensure global theoretical regularity. The working paper also report the log likelihood values for both the unconstrained and constrained models. By comparing these log likelihood values, we see that the imposition of the concavity constraints has not much influence on the flexibility of the NQ expenditure model as the log likelihood values in most cases decrease only slightly. This means that the constrained NQ model can guarantee inference consistent with theory, without compromising much of the flexibility of the functional form.

6.1 Residential Sector

The diagonal entries in Tables 1–3 are the Allen own-price elasticities, σ_{ii}^{a}, and the off-diagonal ones are the (asymmetrical) Morishima elasticities of substitution, σ_{ij}^{m}, calculated using (10). Clearly, the Allen own-price elasticities, σ_{ii}^{a}, are all negative (as predicted by the theory) and highly significant, providing support for the inferred conclusions as to the substitutability/complementarity relationships among the energy goods, based on the Morishima elasticities of substitution. The large estimates (in absolute terms) of the Allen own-price elasticities of substitution for light fuel oil (such as, for example, $\sigma_{ff}^{a} = -81.972$ with a *p*-value of .000 in the case of British Columbia) are probably due to the small expenditure share of light fuel oil.

Figure 5: Average Natural Gas Prices: Residential Sectors



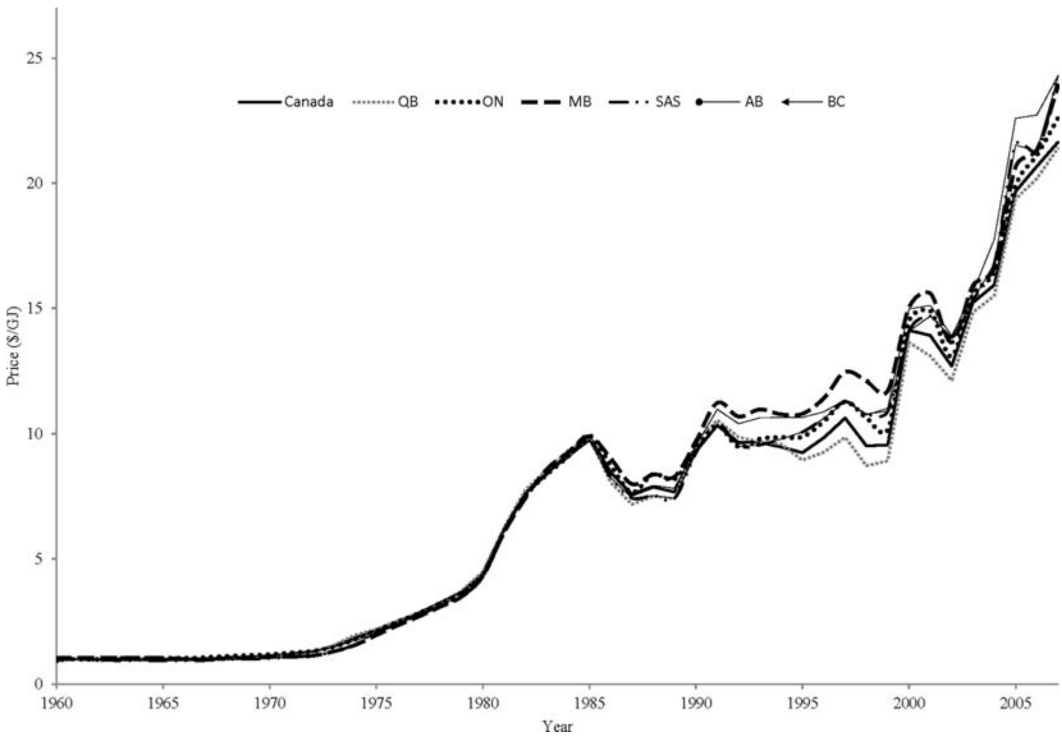
Turning now to the Morishima elasticities of substitution, as can be seen in Table 1, in general they are positive and significant, suggesting substitutability among the energy goods in the residential sector of Canada and the provinces. In particular, the Morishima elasticities of substitution between electricity, e, and natural gas, g, σ^m_{eg} and σ^m_{ge}, are consistently positive for all provinces, irrespective of whether the price of electricity or that of natural gas changes. σ^m_{ge} is negative for Canada as a whole ($\sigma^m_{ge} = -.145$ with a p-value of .000), indicating that electricity and natural gas are Morishima complements when the price of electricity changes. Similarly, electricity, e, and light fuel oil, f, are Morishima substitutes (irrespective of whether the price of electricity or the price of light fuel oil changes) for all provinces and Canada. Finally, natural gas, g, and light fuel oil, f, are Morishima substitutes and only when the price of light fuel oil changes, except in the case of Manitoba ($\sigma^m_{gf} = -.134$ with a p-value of .422), Alberta ($\sigma^m_{gf} = -1.244$ with a p-value of .000), and British Columbia ($\sigma^m_{gf} = -.194$ with a p-value of .318).

Overall, we see that light fuel oil is a strong Morishima substitute for each of electricity and natural gas, while natural gas and electricity are strong substitutes in Alberta ($\sigma^m_{ge} = 1.492$ with a p-value of .000 and $\sigma^m_{eg} = 1.560$ with a p-value of .000) and Saskatchewan ($\sigma^m_{ge} = 1.270$ with a p-value of .000 and $\sigma^m_{eg} = 1.235$ with a p-value of .000), reflecting the large consumption of natural gas in those provinces.

6.2 Commercial Sector

The commercial sector estimates of the Allen own-price elasticities and the Morishima elasticities of substitution are reported in Table 2 in the same fashion as those for the residential

Figure 6: Average Light Fuel Oil Prices: Residential Sectors



sector in Table 1. In the case of Canada as a whole, Manitoba and British Columbia, the regularity conditions are satisfied in the unconstrained version of the model (see Appendix Tables A8, A11 and A14, respectively, in the working paper version of this article) whereas for the rest of the provinces they have to be imposed to achieve theoretical regularity.

We expect the Allen own-price elasticities (the diagonal entries in Table 2), σ_{ii}^{a}, to be negative and this expectation is achieved. However, because as already noted the Allen elasticity of substitution produces ambiguous results off diagonal, we use the Morishima elasticities of substitution to investigate the substitutability/complementarity relation between the energy goods. Based on the asymmetrical Morishima elasticities of substitution, as documented in Table 2, the energy goods are Morishima substitutes with only four of these elasticities being negative which are not statistically different than zero: σ_{ge}^{m} in the case of Quebec and Ontario ($\sigma_{ge}^{m} = -.008$ with a p-value of .463 and $\sigma_{ge}^{m} = -.003$ with a p-value of .678, respectively) and σ_{fg}^{m} in the case of Saskatchewan ($\sigma_{fg}^{m} = -.001$ with a p-value of .974), and σ_{gf}^{m} in the case of Alberta ($\sigma_{gf}^{m} = -.003$ with a p-value of .668, respectively).

Overall, the energy goods are not very strong substitutes or complements in the commercial sectors. Light fuel oil is a moderate substitute for electricity and natural gas in Canada as a whole, Quebec, and Ontario regardless of which energy good price is changing. All energy goods are substitutes, except Quebec and Ontario that show a potential of complementarity between electricity and natural gas when the price of electricity changes; in these cases elasticities are not significantly different than zero.

Table 1: Residential Sector Allen Own-Price Elasticities and Morishima Elasticities of Substitution

		Morishima elasticities of substitution	
Factor *i*	σ^m_{ie}	σ^m_{ig}	σ^m_{if}
Canada			
e	−2.863 (.000)	.326 (.000)	2.034 (.000)
g	−.145 (.000)	−1.542 (.000)	.826 (.000)
f	3.262 (.000)	2.690 (.000)	−7.965 (.000)
Quebec			
e	−1.000 (000)	.315 (.315)	.730 (.361)
g	1.404 (.003)	−7.824 (.000)	2.560 (.038)
f	1.358 (.145)	1.724 (.073)	−20.035 (.052)
Ontario			
e	−1.365 (.000)	.251 (.003)	2.919 (.000)
g	.528 (.000)	−1.482 (.000)	.000 (.998)
f	2.795 (.000)	2.416 (.000)	−55.892 (.000)
Manitoba			
e	−1.283 (.000)	.320 (.008)	1.834 (.000)
g	.287 (.015)	−1.466 (.000)	−.134 (.422)
f	.267 (.002)	.189 (.003)	−10.764 (.000)
Saskatchewan			
e	−4.181 (.000)	1.235 (.000)	2.170 (.000)
g	1.270 (.000)	−1.564 (.000)	.305 (.016)
f	.331 (.000)	.126 (.000)	−4.194 (.000)
Alberta			
e	−7.223 (.000)	1.560 (.000)	3.848 (.000)
g	1.492 (.000)	−1.374 (.000)	−1.244 (.000)
f	.687 (.000)	.396 (.000)	−23.414 (.000)
British Columbia			
e	−1.030 (.000)	.026 (.156)	1.321 (.000)
g	.014 (.389)	−1.030 (.000)	−.194 (.318)
f	.918 (.000)	.889 (.001)	−81.972 (.000)

Note: Sample period, annual data 1960–2007 (T = 48). '*e*' denotes electricity, '*g*' natural gas, and '*f*' light fuel oil. Numbers in parentheses are *p*-values. Numbers in the diagonals are Allen own-price elasticities.

6.3 Industrial Sector

In the industrial sector, in the case of Canada and Manitoba the regularity conditions are satisfied in the unconstrained version of the model (see Appendix Table A15 and A18 in the working paper version of this article). In all other cases the regularity conditions had to be imposed to achieve theoretical regularity.

The industrial sector estimates of the Allen own-price elasticities and the Morishima elasticities of substitution are reported in Table 3. As can be seen, as in the residential and commercial sectors, the Allen own-price elasticities of substitution are all negative. The Morishima elasticities of substitution are all positive for Canada as a whole, Manitoba, Saskatchewan, Alberta, and British Columbia, indicating substitutability among electricity, natural gas, and light fuel oil. Similar to the commercial sector results, electricity and natural gas are complements when the price of electricity changes in Quebec and Ontario. Moreover, all energy goods show weak substitutability in British Columbia. The weak evidence of interfuel substitution in the industrial sector can be explained by the fact that this sector includes a wide variety of machinery with different fuel inputs that do not let for interfuel substitutability.

Table 2: Commercial Sector Allen Own-Price Elasticities and Morishima Elasticities of Substitution

	Morishima elasticities of substitution		
Factor *i*	σ^m_{ie}	σ^m_{ig}	σ^m_{if}
Canada			
e	−.003 (.000)	.011 (.074)	.043 (.000)
g	.009 (.002)	−.004 (.004)	.045 (.000)
f	.025 (.000)	.029 (.000)	−.008 (.000)
Quebec			
e	−.002 (.033)	.018 (.470)	.090 (.003)
g	−.008 (.463)	−.009 (.121)	.115 (.010)
f	.028 (.001)	.080 (.051)	−.019 (.005)
Ontario			
e	−.002 (.257)	.005 (.616)	.050 (.000)
g	−.003 (.678)	−.004 (.069)	.059 (.000)
f	.019 (.010)	.036 (.006)	−.011 (.000)
Manitoba			
e	−.000 (.226)	.002 (.682)	.014 (.000)
g	−.000 (.890)	−.001 (.231)	.016 (.000)
f	.005 (.000)	.010 (.056)	−.003 (.000)
Saskatchewan			
e	−.002 (.111)	.004 (.834)	.007 (.000)
g	.010 (.353)	−.000 (.930)	.001 (.922)
f	.012 (.002)	−.001 (.974)	−.001 (.502)
Alberta			
e	−.008 (.000)	.049 (.002)	.004 (.039)
g	.056 (.000)	−.006 (.010)	−.003 (.668)
f	.033 (.000)	.020 (.081)	−.000 (.871)
British Columbia			
e	−.003 (.000)	.016 (.001)	.016 (.000)
g	.013 (.000)	−.004 (.001)	.019 (.000)
f	.013 (.000)	.019 (.000)	−.004 (.000)

Note: Sample period, annual data 1960–2007 (T = 48). '*e*' denotes electricity, '*g*' natural gas, and '*f*' light fuel oil. Numbers in parentheses are *p*-values. Numbers in the diagonals are Allen own-price elasticities.

7. COMPARISON WITH OTHER STUDIES

It is difficult to provide a comparison between our results and those obtained in analyses performed by others. As we mentioned in the introduction, the major contributions in this area are quite outdated and, as can be seen in Table 4, most of the studies have employed the translog functional form introduced by Christensen et al. (1975). Although the translog provides arbitrary elasticity estimates at the point of approximation (i.e. locally), there is evidence that this model fails to meet the theoretical regularity conditions of neoclassical microeconomic theory (positivity, monotonicity and curvature) in large regions. In this regard, as Barnett (2002, p. 199) put it, "without satisfaction of both curvature and monotonicity, the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid."

In fact, most of the (interfuel substitution) studies listed in Table 4 do not produce inference consistent with neoclassical microeconomic theory, and do not even report the results of full regularity checks, except for the recent studies by Serletis et al. (2010, 2011). Moreover, as can be

Table 3: Industrial Sector Allen Own-Price Elasticities and Morishima Elasticities of Substitution

	Morishima elasticities of substitution		
Factor i	σ^m_{ie}	σ^m_{ig}	σ^m_{if}
Canada			
e	−.001 (.055)	.007 (.374)	.016 (.000)
g	.011 (.187)	−.001 (.260)	.011 (.000)
f	.015 (.006)	.007 (.180)	−.001 (.000)
Quebec			
e	−.000 (.850)	.003 (.888)	.019 (.189)
g	−.002 (.905)	−.000 (.706)	.024 (.261)
f	.006 (.670)	.016 (.578)	−.000 (.168)
Ontario			
e	−.000 (.909)	.008 (.582)	.019 (.000)
g	−.002 (.862)	−.002 (.245)	.029 (.000)
f	.004 (.459)	.023 (.070)	−.003 (.000)
Manitoba			
e	−.005 (.000)	.038 (.000)	.009 (.000)
g	.043 (.000)	−.004 (.000)	.003 (.116)
f	.029 (.000)	.018 (.001)	−.001 (.000)
Saskatchewan			
e	−.006 (.000)	.058 (.000)	.000 (.895)
g	.059 (.000)	−.006 (.000)	−.000 (.939)
f	.030 (.000)	.029 (.000)	−.000 (.999)
Alberta			
e	−.002 (.094)	.018 (.107)	.001 (.444)
g	.019 (.098)	−.002 (.114)	−.001 (.586)
f	.011 (.089)	.008 (.131)	−.000 (.948)
British Columbia			
e	−.000 (.955)	.001 (.925)	.004 (.000)
g	−.000 (.969)	−.000 (.733)	.006 (.000)
f	.001 (.818)	.004 (.391)	−.000 (.000)

Note: Sample period, annual data 1960–2007 (T = 48). 'e' denotes electricity, 'g' natural gas, and 'f' light fuel oil. Numbers in parentheses are p-values. Numbers in the diagonals are Allen own-price elasticities.

seen under the 'Data' column of Table 4, most of these studies investigate energy demand in industrial sectors and also use different energy goods (see, for example, the 'Goods' column of Table 4). Finally, none of these studies reports Morishima elasticities of substitution, which are the correct measures of substitution elasticities, as noted by Blackorby and Russell (1989); they only report income and own- and cross-price elasticities.

Our results are comparable to those reported by Serletis et al. (2010, 2011) who investigate sectoral interfuel (oil, natural gas, coal, and electricity) substitution, using the NQ cost function, treating curvature as the maintained hypothesis, as we do in this paper. Overall, our energy demand elasticities represent important market parameters and highlight the fact that the substitution between different energy inputs has been quite restricted. These energy demand elasticities could be used to investigate how energy prices will change with economic fluctuations and how taxes and subsidies will affect the level of economic activity.

8. CONCLUSION

We focus on the aggregate demand for electricity, natural gas, and light fuel oil in Canada as a whole and six of its provinces—Quebec, Ontario, Manitoba, Saskatchewan, Alberta, and British

Table 4: A Summary of Flexible Functional Forms Interfuel Substitution Studies

Study	Model used	Data	Goods	Curvature imposed
Berndt and Wood (1975)	Translog	Annual U.S.—Time series data—Industrial sector (1947–1971)	Labour, capital, materials, and aggregate energy	No Checked it, and is satisfied at all points
Fuss(1977)	Translog	Annual five regions of Canada—Pool of time series and cross-section—Industrial sector (1961–71)	Labour, capital, materials, and energy (coal, LPG, fuel oil, natural gas, electricity, and motor gasoline)	No
Halvorsen (1977)	Translog	Annual U.S.—Cross-section data—Industrial sector (1971)	Coal, natural gas, electricity, and fuel oil	No Tested it, but is not satisfied at all points
Pindyck (1979)	Translog	Annual ten developed countries including Canada—Pool of time series and cross-section—Industrial sector (1959–73)	Labour, capital, materials, and energy (coal, natural gas, electricity, and fuel oil)	No
Hall (1986)	Translog	Annual seven OECD countries including Canada—Time series data—Industrial sector (1960–79)	Petroleum products, natural gas, coal, and electricity	No
Considine (1989)	Translog and Linear Logit	Annual U.S.—Time series data—Industrial sector (1970–1985)	Petroleum products (residual fuel, distillate fuel, and kerosene), natural gas, coal, and electricity	No Tested it, but is not satisfied at all points
Jones (1995)	Translog and Linear Logit	Annual U.S.—Time series data—Industrial sector (1960–1992)	Petroleum products (residual fuel, distillate fuel, and kerosene), natural gas, coal, and electricity	No Checked it, but is not satisfied at all points
Serletis et al. (2010)	NQ	Annual 15 countries—Time series data—Residential, Industrial, Electricity generation, and Transportation sectors (1980–2006)	Oil, natural gas, coal, and electricity	Yes Satisfied at all points
Serletis et al. (2011)	NQ	Annual 15 countries—Pooled data—Industrial sector (1980–2006)	Oil, natural gas, coal, and electricity	Yes Satisfied at all points

Columbia—in the residential, commercial, and industrial sectors. We use annual data, over the period from 1960 to 2007 (a total of 48 observations), and employ the NQ cost function, introduced by Diewert and Wales (1987), to investigate energy demand in the commercial and industrial sectors and (for the first time in the energy demand literature) the locally flexible normalized quadratic (NQ) expenditure function, introduced by Diewert and Wales (1988) to investigate energy demand in the residential sector. We treat the concavity property as a maintained hypothesis, using methods developed by Diewert and Wales (1987, 1988), and provide evidence consistent with neoclassical microeconomic theory.

We provide a full set of elasticities—expenditure elasticities, own- and cross-price elasticities, Allen own- and cross-price elasticities of substitution, and the asymmetrical Morishima elasticities of substitution. The expenditure elasticities reveal that in general the energy goods are normal goods, except for light fuel oil which is an inferior good in some of the provinces and sectors. We find that the interfuel Morishima elasticities of substitution (the correct measures of substitution when there are more than two goods) are in general positive and statistically significant. Moreover, our results indicate limited substitutability between electricity and natural gas, but strong substitutability between light fuel oil and each of electricity and natural gas in most cases. We also find that the residential sector reveals a higher potential for substitution between energy goods in all provinces than the commercial and industrial sectors.

Our results are consistent with the evidence reported by Serletis et al. (2010, 2011) who investigate sectoral interfuel (crude oil, natural gas, coal, and electricity) substitution, using the NQ cost function (not the NQ expenditure function) and time series data for a number of OECD and non-OECD countries (including China and India). As Serletis et al. (2010, p. 27) put it, the "results highlight the fact that the substitution between different energy inputs has been quite restricted, suggesting that fossil fuels will continue to maintain their major role as a source of energy in the near future. Therefore, such daunting tasks as curbing carbon emissions and preventing climate change require a more active and focused energy policy. Also, because interfuel substitution is limited in the near term, there will be a greater need for relative price changes to induce switching to a lower carbon economy."

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