Energy Sector Innovation and Growth: An Optimal Energy Crisis

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ABSTRACT

We study the optimal transition from fossil fuels to renewable energy in a neoclassical growth economy with endogenous technological progress in energy production. Innovations keep fossil energy costs under control even as increased exploitation raises mining costs. Nevertheless, the economy transitions to renewable energy after about 80% of available fossil fuels are exploited. The energy shadow price remains more than double current values for over 75 years around the switch time. Consumption and output growth decline sharply during the transition period, which we thus identify as an "energy crisis." The model highlights the important role energy can play in influencing economic growth.

Keywords: Energy innovation, Energy transition, Energy cost, Economic growth

http://dx.doi.org/10.5547/01956574.37.1.phar

1. INTRODUCTION

Since the days of the industrial revolution, economic growth has been powered largely by fossil fuel. In recent decades, large-scale energy production from renewable sources has become technologically feasible, albeit expensive and as yet uncompetitive without subsidies. Nevertheless, since fossil fuel is a finite resource that will become more scarce, non-fossil energy must eventually predominate. Admittedly, technological progress can moderate fossil fuel cost increases by expanding the quantity of economically viable resources, as illustrated most recently by natural gas and oil production from shales. Technological progress in the form of improved energy efficiency also can reduce the amount of fuel needed to provide a given level of energy services. Nevertheless, expanding energy demand resulting from economic and population growth implies that fossil fuel costs ultimately will rise.[1]

The need to transition to more expensive alternatives to fossil fuels is likely to impose substantial costs. It is often argued that these costs are sub-optimal and should, if possible, be reduced via appropriate policies.[2]

1. DOE/NETL (2007) summarizes several forecasts for the likely year of peak conventional oil production.

2. See, for example, Farrell and Brandt (2006). The possible macroeconomic costs of an energy transition are distinct from any environmental or congestion externalities associated with using different energy sources. We do not discuss the latter in this paper. In addition, since our model is long run in nature and does not involve uncertainty, we do not model temporary energy crises due to unanticipated supply or demand shocks and binding short-run production capacity constraints.

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We investigate this claim by studying the transition from fossil fuels to renewable energy in a simple neoclassical growth economy in which energy is needed to produce the economy's single consumption good and there is endogenous technological progress in *both* fossil and renewable energy technologies. In the case of fossil fuels, investments in new technologies can offset the increase in mining costs that result from cumulative resource development.[3] In the case of renewables, accumulated knowledge resulting from use and direct R&D investment lowers unit production cost until a technological limit is attained. Energy services supplied by fossil fuels or renewable sources are assumed to be perfect substitutes.[4] We show that an "energy crisis" around the time the economy optimally abandons fossil fuels can be *efficient*.

The model gives rise to several different regimes, which are depicted graphically in Figure 2 below. Initially, growth occurs through the use of fossil fuel while investment in fossil fuel technology keeps energy costs from rising substantially.[5] However, fossil energy investments, which must be made at an increasing rate to keep costs under control as resources are depleted, eventually cease. Fossil fuels then become uncompetitive and renewable energy powers the economy. Interestingly, the transition from fossil fuels to renewable energy must occur when the cost of fossil energy is less than the initial cost of renewable energy. The reason is that the learning by doing element of renewable energy production lowers the shadow price (or full cost) of renewable energy, making it worthwhile to transition before the explicit cost of fossil energy reaches the initial explicit cost of renewable energy.

Once the economy shifts to renewable energy, learning by doing and R&D investment reduce renewable energy cost until a technological frontier is reached. A constant cost of renewable energy then transforms the model into a simple endogenous growth model that can be solved analytically. Since the regime occurs so far into the future, different assumptions about the limiting renewable efficiency have only trivial effects on the solution up to the transition between energy sources, which is the primary focus of the paper.

After characterizing the optimal path qualitatively, we solve the model numerically by calibrating the parameters and initial values of the endogenous variables to match the global economy in 2004. The main data source we use is the *GTAP 7 Data Base* produced by the *Center for Global Trade Analysis* in the Department of Agricultural Economics, Purdue University. The GTAP data is most useful for our purpose since it provides a consistent set of international macroeconomic accounts that also take account of energy flows.

We find that per capita consumption grows more slowly than per capita output in the fossil energy regime. The rising cost of energy, and rising investment in fossil fuel technologies, both take increasing resources away from consumption. Toward the end of the fossil regime, optimal

3. The mining technology variable can also be thought of as a reduced form means of capturing the effect of energy efficiency improvements. By reducing the resource input needed to provide a given level of energy services, efficiency improvements also slow down the rise in costs from resource depletion.

4. Fossil and renewable energy sources therefore are not employed at the same time in our model. While this implication might at first seem counter-factual, the coexistence of technologies that produce energy at higher cost is largely due to subsidies, which are absent in our analysis.

5. Short-run energy price spikes result more from supply and demand shocks in the presence of production capacity that is fixed in the short run than from longer-run depletion. Energy prices also fluctuate more in reality than in our model because we have assumed gradual technological progress rather than periodic breakthroughs amidst steady improvements as seen in practice.

investments in fossil fuel technologies become rather large. In addition, the real cost of energy peaks at the switch time and is more than double current levels for over 75 years around the switch time. These other resource demands constrain final consumption and investment in capital. In particular, while the consumption share of output remains close to its current value of around 60% for the first fifty years of the fossil fuel regime, it plunges to well below 40% of output at the switch date. Thus, even though the energy transition path is efficient, our model predicts an "energy crisis" especially in the lead up to, and around the time of, the transition between energy sources.

After the transition to renewable energy, the declining cost of energy allows consumption to grow faster than output. Nevertheless, the cost of renewable energy remains high for a long time, while optimal investments in renewable technology also tend to be relatively large immediately following the transition to renewable sources. These factors prevent the consumption share from rising back above 55% of output for another 150 or so years.

Finally, our analysis emphasizes the importance of modeling progress in fossil, as well as in renewable energy production in policy discussions regarding subsidizing renewable energy sources. Advances such as shale oil and gas, oil sands production in Canada, and deep water exploration, increase the supply of fossil fuel and imply that the "parity cost target" for renewables is a moving one. In our quantitative analysis, technological advancements allow fossil fuels to remain competitive for longer than is commonly assumed. Ultimately, about 80% of the technically recoverable fossil fuel resource is exploited, with the transition to renewable energy occurring toward the end of this century.

2. RELATED LITERATURE

Our approach is related to a number of papers in the literature. Parente (1994) studies a model in which firms adopt new technologies as they gain firm-specific expertise through learning by doing. He identifies conditions under which equilibria in his model exhibit constant growth of per capita output. As in most of the literature on economic growth, Parente abstracts from issues related to energy.

Chakravorty et al. (1997) develop a model with substitution between energy sources, improvements in extraction, and a declining cost of renewable energy. They find that if historical rates of cost reductions in renewables continue, a transition to renewable energy will occur before over 90% of the world's coal is used. Our model is complementary to theirs. By modeling investment in energy technologies, we generate an endogenous transition to renewable energy, while allowing for investment in physical capital enables us to explore the endogenous trade-off between the cost of energy and economic growth. Unlike Chakravorty et al. we do not study the implications of energy use for carbon dioxide emissions[6] and we do not conduct policy experiments.

Tsura and Zemelb (2003) analyze how learning through R&D affects the optimal transition from nonrenewable energy to a backstop substitute. They find that, if the initial knowledge level is sufficiently low, R&D should start as early as possible and at the highest affordable rate. Our analysis differs in many respects, including using a general equilibrium as opposed to a partial equilibrium model and allowing for progress in fossil fuel technologies.

6. While we do not explicitly discuss environmental externalities associated with energy use, allocations in our model can be interpreted as laissez-faire or "business as usual" scenarios in models where such environmental externalities are included.

More recently, Golosov et al. (2014) built a macroeconomic model that incorporates energy use and the resulting environmental consequences. They derive a formula describing the optimal tax due to the externality from emissions and provide numerical values for the size of the tax in a calibrated version of their model. However, they abstract from endogenous technological progress in either fossil fuels or renewables. As a result, transitions between different energy regimes are exogenous in their model.

Van der Ploeg and Withagen (2012) use a growth model to investigate the possibility of a green paradox, that is a tendency for promotion of renewable energy to accelerate the exploitation of fossil fuel by lowering resource rents and thus the opportunity cost of extraction. Van der Ploeg and Withagen (2014) study optimal climate policy in a Ramsey growth model with exhaustible oil reserves and an infinitely elastic supply of renewables. They consider climate change issues and characterize the different energy regimes, as well as the optimal carbon tax along the economy's growth path. Our model differs from their studies primarily by allowing for technological progress in fossil fuels. We also calibrate our model using world-economy data.

Finally, Acemoglu et al. (2012) study a growth model that takes into consideration the environmental impact of operating "dirty" technologies. They examine the effects of policies that tax innovation and production in the dirty sectors. Their paper focuses on long run growth and sustainability and abstracts from the endogenous evolution of R&D expenditures. They find that subsidizing research in the "clean" sectors can speed up environmentally friendly innovation while avoiding the negative impact of taxes or quantitative emission controls on economic growth. Optimal behavior in their model requires an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production.

Our work differs from Acemoglu et al. (2012) by explicitly connecting R&D, energy, and growth, and by focusing on the effects of the energy transition on growth rather than environmental issues. The transition between energy sources in the two models is very different primarily because we model technological progress in *both* the renewable and the fossil fuel sectors. More generally, most of the literature ignores the key idea that advances in fossil fuel extraction and end-use efficiency technologies are of first-order importance in addressing the energy transition question.

3. THE MODEL

Letting $c(t)$ denote per capita consumption of the single consumption good in the economy at time t,[7]the objective is to maximize the lifetime present value of utility of a representative agent. In common with much of the growth literature, we assume that the instantaneous utility takes the constant relative risk averse form, so the objective becomes:

$$U = \int_0^\infty e^{-\beta\tau} \frac{c(\tau)^{1-\gamma}}{1-\gamma} d\tau \tag{1}$$

where $e^{-\beta\tau}$ is the discount factor and γ is the coefficient of relative risk aversion.[8]

Per capita output y can be produced using per capita capital k and energy[9] E as inputs.

7. We model economic activity in continuous time, indexed by t. The state variables, the controls, and the technology variables thus are functions of t. Henceforth, we shall often simplify notation by omitting time as an explicit argument.

8. More precisely, since there is no uncertainty in our model, this parameter relates to intertemporal elasticity of substitution.

9. Although "energy" is more properly thought of as an input into the energy sector, and "energy services" an output, we use the terms interchangeably.

Ignoring for the moment the required energy input, we assume that output depends linearly on k. Effectively, this allows technological progress to expand labor input through investment in human capital even if hours and number of employees remain fixed. Hence, the marginal product of capital does not decline as k accumulates. Capital depreciates at the rate δ, while investment in new capital is denoted by i:

$$\dot{k} = i - \delta k \tag{2}$$

Energy is also an essential input to production. We assume for simplicity that there is no substitution between energy and non-energy inputs in producing y,[10] allowing the production function to be written $y = \min\{Ak, E\}$. Since it is costly to produce both k and E, however, it will be optimal at all times to have $y = Ak = E$.

Energy can be supplied from two different sources. Denote the per capita energy derived from fossil fuel resources by $R \geq 0$. We assume that per capita renewable energy supply $B \geq 0$ is a perfect substitute for the energy produced from fossil fuels.[11] Thus, we must have $E = R + B$ and the optimal solution will have at all times:

$$y = Ak \tag{3}$$

and

$$R + B = y \tag{4}$$

3.1 Fossil Fuel Supply

Higher population and per capita economic growth rates will increase the rate of depletion of fossil fuels. Letting Q denote the population, growing (exogenously) at rate π, current fossil fuel use will be QR, and the amount used to date, S, will be the integral of QR:

$$\dot{S} = QR \tag{5}$$

We assume the cost of fossil fuel production has two components. This is somewhat analogous to the distinction Venables (2014) draws between costs of extraction on the intensive margin on the one hand, versus costs of new field development, or expansion on the extensive margin, on the other hand. However, we simplify by assuming that, for a given value of S, resources can be extracted at a constant marginal (and average) cost. Depletion (an increase in S) raises that cost over time,[12] but technological progress can offset the cost increases. The state of technical

10. Allowing for investment in end-use energy efficiency would require an additional state, and corresponding co-state, variable, considerably complicating the numerical analysis without adding much to the issues under discussion. In particular, as noted above, technological progress in energy production can also be considered as capturing a reduced need for energy input, and a lower per-unit cost of energy supply, as end-use energy efficiency increases. Effectively, this amounts to defining energy supplies in efficiency units.

11. This assumption is admittedly extreme and it is mainly adopted for simplicity. Using a continuity argument, we can show that our results remain true if the degree of substitutability is high, but not perfect. See Hassler et al. (2011) for a discussion of desirable short and long run substitution elasticities in this context.

12. Heal (1976) introduced the idea of an increasing marginal cost of extraction to show that the optimal price of an exhaustible resource begins above marginal cost, and falls toward it over time. This claim is rigorously proved in Oren and Powell (1985). See also Solow and Wan (1976).

knowledge about producing energy services from fossil fuels is encapsulated in a variable N, which does not depreciate over time, and where the chosen investment n leads to an accumulation of N:

$$\dot{N} = n \tag{6}$$

Investment n could be associated with bringing new fields into production as emphasized by Venables (2014). However, we have in mind longer-run processes, such as new technologies that enable exploitation of new categories of resources (shale gas and oil, deepwater or pre-salt deposits, oil sands, oil shale, methane hydrates and underground gasification of deep coal), or increase the efficiency with which fossil fuel is used to provide useful energy services.[13] While the total feasible technically recoverable fossil energy resource $\bar{S}$ is vast, in the absence of investment in N, we assume that the maximum recoverable resource S_0 is far smaller. Furthermore, our model predicts that the transition to renewable energy will take place well before S reaches $\bar{S}$ because the cost increases make fossil fuels uncompetitive.

Specifically, we assume that if N were to remain at zero, the marginal cost of supplying a unit of fossil energy services, $g(S,0)$ would be increasing and convex in S and unbounded as $S \uparrow S_0$. On the other hand, if N were to increase to infinity the upper bound on S (where $g(S,N) \uparrow \infty$) would converge to $\bar{S}$. A simple functional form that incorporates these assumptions, which is illustrated in Figure 1, is:

$$g(S,N) = \alpha_0 + \frac{\alpha_1}{\bar{S} - S - \alpha_2/(\alpha_3 + N)} = \alpha_0 + \frac{\alpha_1(\alpha_3 + N)}{(\bar{S} - S)(\alpha_3 + N) - \alpha_2} \tag{7}$$

The terms $\alpha_0, \alpha_1, \alpha_2$ and α_3 in (7) are parameters, and the afore-mentioned S_0 is $\bar{S} - \alpha_2/\alpha_3$. Investment in new technologies expands the temporary capacity limit, and the flat portion of the marginal cost curve, to the right, extending the competitiveness of fossil fuels.[14]

It is straight-forward to show that $\partial g/\partial S > 0$, $\partial^2 g/\partial S^2 > 0$, $\partial g/\partial N < 0$, $\partial^2 g/\partial N^2 > 0$ and $\partial^2 g/\partial N \partial S < 0$. Thus, cumulative exploitation S increases current marginal cost g at an increasing rate, while investment in fossil fuel technology N decreases g at a decreasing rate. Also, investment in N delays the increase in costs resulting from depletion.

For energy to be productive on net, we need the value of output produced from energy input to exceed the costs of producing that energy input. Thus, whenever fossil fuel is used to provide energy input, we must have $1 > g(S,N)$. Function (7) implies that this constraint eventually must be violated as exhaustion of fossil fuel resources increases $g(S,N)$.

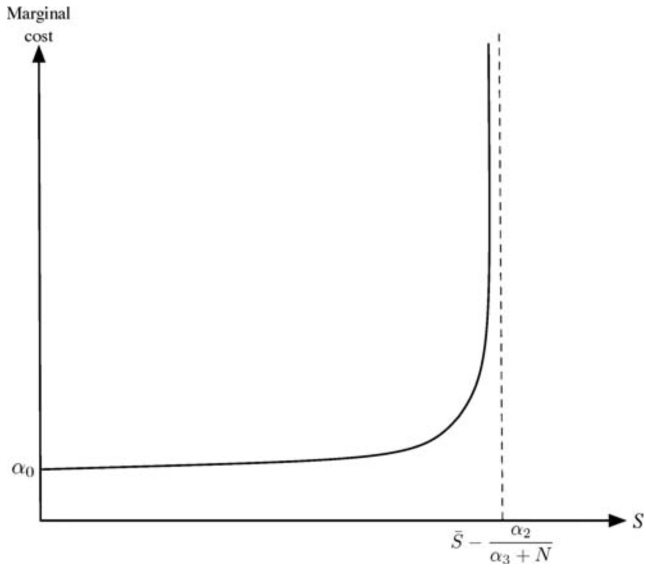
3.2 Renewable Energy Technologies

The renewable technology combines some output (effectively, capital) with a non-depleteable energy source (for example, sunlight, wind, waves or stored water) to produce more useful

13. Since we have defined the energy supplies in efficiency units, improvements in energy efficiency also reduce the per-unit cost of supplying an additional unit of energy services R.

14. In contrast to the renewable sector, we do not assume the fossil fuel industry experiences cost reductions through learning by doing. Learning by doing in accumulating N would effectively increase the productivity of investments n. Depletion can be viewed as "inverse learning by doing," since cumulative past production raises current costs. Investment in N tends to offset this process in the case of fossil fuels, whereas investment in R&D reinforces the cost-reducing effects of learning by doing in the renewable sector.

Figure 1: Marginal Cost of Energy from Fossil Fuels



output than has been used as an input. Explicitly, using p to denote the marginal cost of the energy services produced using the renewable technology, we require $p < 1$.

Technological progress reduces p as knowledge accumulates until p attains a lower limit, Γ_2, determined by physical constraints. Explicitly, using H to denote the stock of renewable energy production knowledge, and $\Gamma_1^{-\alpha}$ the initial value of p (when $H = 0$), we assume:[15]

$$p = \begin{cases} (\Gamma_1 + H)^{-\alpha} & \text{if } H \le \Gamma_2^{-1/\alpha} - \Gamma_1, \\ \Gamma_2 & \text{otherwise} \end{cases} \tag{8}$$

for parameters Γ_1, Γ_2 and α, with $\Gamma_1^{-\alpha} > g(0,0)$ so renewable energy is initially uncompetitive with fossil fuels.

We assume a two-factor learning model, whereby direct R&D expenditure j can accelerate the accumulation of knowledge about the renewable technology arising from its use:[16]

$$\dot{H} = \begin{cases} B^{\psi} j^{1-\psi} & \text{if } H \le \Gamma_2^{-1/\alpha} - \Gamma_1, \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

In particular, once H reaches its upper limit, further investment in the technology would be worthless and we should have $j = 0$. The parameter ψ determines how investment in research enhances the accumulation of knowledge from experience. Klaassen et. al. (2005) derive robust estimates suggesting that direct R&D is roughly twice as productive for reducing costs as is learning by doing.[17] Hence, we assume that $\psi = 0.33$.

15. The functional form is from literature on learning curves, such as International Energy Agency (2000).

16. The Cobb-Douglas form of (9) implies that research alone cannot reduce the costs of renewable energy production. Experience in deploying the technologies also is essential.

17. Following Kouvaritakis et al. (2000), Klaassen et. al. (2005) estimated a two-factor learning model that allowed both capacity expansion and direct public R&D to reduce costs of wind turbine farms in Denmark, Germany and the UK. They claim their results support the two-factor learning curve formulation.

3.3 The Optimization Problem

Goods are consumed, invested in k, N, or H, or used for producing fossil fuel or renewable energy input. This leads to a resource constraint (in per capita terms):

$$c + i + j + n + g(S,N)R + pB = y \tag{10}$$

The objective function (1) is maximized subject to the differential constraints (2), (5), (6) and (9) with initial conditions $S(0) = N(0) = 0$, $k(0) = k_0 > 0$ and $H(0) = 0$, the resource constraint (10), the definitions of output (3), energy input (4) and the evolution of the cost of renewable energy supply (8). The control variables are c, i, j, R, n and B, while the state variables are k, H, S and N. Denote the corresponding co-state variables by q, η, σ and v. Let λ be the Lagrange multiplier on the resource constraint and ϵ the multiplier on the energy constraint (the shadow price of energy). To allow for either type of energy to be unused, and for investment in either technology to be zero, let μ be the multiplier on $j \geq 0$, ω the multiplier on $n \geq 0$, ξ the multiplier on $R \geq 0$ and ζ the multiplier on $B \geq 0$. Finally, let χ be the multiplier on the constraint $H \leq \Gamma_2^{-1/\alpha} - \Gamma_1$.

Define the current value Hamiltonian and thus Lagrangian by

$$\begin{aligned} \mathcal{H} = \frac{c^{1-\gamma}}{1-\gamma} + \lambda[Ak - c - i - j - n - g(S,N)R - (\Gamma_1 + H)^{-\alpha}B] + \epsilon(R + B - Ak) + \\ q(i - \delta k) + \eta B^{\psi} j^{1-\psi} + \sigma QR + vn + \mu j + \omega n + \xi R + \zeta B + \chi \Gamma_2^{-1/\alpha} - \Gamma_1 - H] \end{aligned} \tag{11}$$

The first order conditions for a maximum with respect to the control variables are:

$$\frac{\partial \mathcal{H}}{\partial c} = c^{-\gamma} - \lambda = 0 \tag{12}$$

$$\frac{\partial \mathcal{H}}{\partial i} = -\lambda + q = 0 \tag{13}$$

$$\frac{\partial \mathcal{H}}{\partial j} = -\lambda + (1-\psi)\eta B^{\psi} j^{-\psi} + \mu = 0;\ \mu j = 0,\ \mu \geq 0,\ j \geq 0 \tag{14}$$

$$\frac{\partial \mathcal{H}}{\partial n} = -\lambda + v + \omega = 0,\ \omega n = 0,\ \omega \geq 0,\ n \geq 0 \tag{15}$$

$$\frac{\partial \mathcal{H}}{\partial R} = -\lambda g(S,N) + \epsilon + \sigma Q + \xi = 0,\ \xi R = 0,\ \xi \geq 0,\ R \geq 0 \tag{16}$$

$$\frac{\partial \mathcal{H}}{\partial B} = -\lambda(\Gamma_1 + H)^{-\alpha} + \epsilon + \eta\psi B^{\psi-1} j^{1-\psi} + \zeta = 0,\ \zeta B = 0,\ \zeta \geq 0,\ B \geq 0 \tag{17}$$

The differential equations for the co-state variables are:

$$\dot{q} = \beta q - \frac{\partial \mathcal{H}}{\partial k} = (\beta + \delta)q - \lambda A + \epsilon A \tag{18}$$

$$\begin{aligned} &\dot{\eta} = \beta\eta - \frac{\partial \mathcal{H}}{\partial H} = \beta\eta - \lambda\alpha(\Gamma_1 + H)^{-\alpha-1}B + \chi; \\ &\chi(\Gamma_2^{-1/\alpha} - \Gamma_1 - H] = 0,\ \chi \geq 0,\ H \leq \Gamma_2^{-1/\alpha} - \Gamma_1 \end{aligned} \tag{19}$$

$$\dot{\sigma} = \beta\sigma - \frac{\partial \mathcal{H}}{\partial S} = \beta\sigma + \lambda\frac{\partial g}{\partial S}R \tag{20}$$

$$\dot{v} = \beta v - \frac{\partial \mathcal{H}}{\partial N} = \beta v + \lambda\frac{\partial g}{\partial N}R \tag{21}$$

We also recover the resource constraint (10) and the differential equations for the state variables, (2), (5), (6) and (9).

3.4 The Evolution of the Economy

Section 1 of the Appendix provides a detailed analysis of the evolution of the economy through various regimes of energy use and energy technology investment working backwards through time. In this section, we provide an overview of the different regimes.

We assume parameter values are set so that initially all energy services are provided by lower cost fossil fuels. As fossil fuels are depleted, however, the shadow price of energy services (ϵ) will rise. Although investments in N moderate the increase, eventually the value of ϵ from (16) will rise to equal the value of ϵ in the renewable regime obtained from (17). At that time, which we will denote T_1, the economy switches to use only renewable energy.

The co-state variable σ corresponding to the state variable S satisfies $\sigma = \partial V/\partial S$, where V denotes the maximized current value of the objective subject to the constraints. In particular, $\sigma = 0$ at T_1 since S has no effect once fossil fuel use ceases. Also, since an increase in S raises fossil fuel cost, $\sigma < 0$ for $t < T_1$.[18] Hence, (16) implies that the shadow price of energy converges to $\epsilon = \lambda g(S,N)$ as $t \to T_1$.

Once renewable energy use begins, the accumulation of experience and explicit R&D investment will raise H. Eventually, however, the economy will attain the technological frontier for renewable energy efficiency at another time T_2. Explicit investment j in H will then cease. Since changes in H have no further effect on maximized utility beyond T_2, the co-state variable η corresponding to H must satisfy $\eta = \partial V/\partial H = 0$ at T_2. For $t \in [T_1, T_2)$, $\eta > 0$ since an increase in H will lower the shadow price of energy services and raise V.[19] In particular, we must have $\eta > 0$ at T_1 with $\eta \downarrow 0$ as $t \to T_2$.

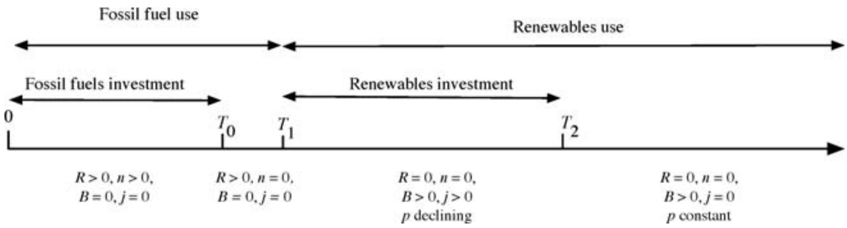
From (14), $B > 0$ implies $j^{\psi}\lambda \geq (1-\psi)\eta B^{\psi} > 0$, and when $j > 0$, it must satisfy

$$j = [(1-\psi)(\eta/\lambda)]^{1/\psi}B \tag{22}$$

18. Formally, if $\sigma(\tau) > 0$ for $\tau < T_1$, since $\partial g/\partial S > 0$, (20) would imply $\dot{\sigma} > 0$ and $\sigma > 0$ for all $t \geq \tau$ contradicting $\sigma(T_1) = 0$.

19. Formally, if $\eta(\tau) \leq 0$ at $\tau \geq T_1$, (19) would imply $\dot{\eta} < 0$ and $\eta < 0$ for all $t \geq \tau$ contradicting $\eta(T_2) = 0$.

Figure 2: Regimes of Energy Use and Investment



implying $B>0$. Thus, we cannot have $j>0$ and $B=0$, and j must become positive for the first time at T_1. It follows that $H=0$ at T_1. Then (17) and continuity of the shadow price of energy at T_1 will require

$$\epsilon = \lambda g(S,N) = \lambda\Gamma_1^{-\alpha} - \eta\psi B^{\psi-1} j^{1-\psi} \tag{23}$$

Since the total energy input requirement $R+B=Ak$, B must jump from 0 to $Ak>0$ (and R from Ak to 0) at T_1. Then $B>0$ at T_1 implies $j>0$. Equation (23) then implies that the transition from fossil fuels to renewable energy will occur when $g(S,N)<\Gamma_1^{-\alpha}$. Thus, the benefits of learning by doing make it worthwhile to transition to renewable energy *before* the cost of fossil fuels reaches parity with the cost of renewable energy.

Finally, since changes in N have no effect beyond T_1, the co-state variable v satisfies $v = \partial V/\partial N = 0$ at T_1. However, (13) implies $\lambda = q > 0$, so from (15), $\omega = \lambda - v > 0$ and hence $n=0$ at T_1. For $t<T_1$, any increases in N will reduce fossil fuel costs and raise the maximized value of the objective subject to the constraints, so $v = \partial V/\partial N > 0$.[20] As we move backwards in time from T_1 while holding N fixed $\partial g/\partial N$ will change rapidly. As a result, v will increase faster than λ until we arrive at a time T_0 when $v=\lambda$. For $t<T_0$, $n>0$ and $v=\lambda$. However, $v<\lambda$ for $t>T_0$, and from (15) investment in fossil fuel technology ceases at T_0 and remains zero thereafter.

In summary, the economy passes through the regimes illustrated in Figure 2. Section 1 of the Appendix discusses the evolution of the endogenous economic and energy system variables in each of these regimes. Since an analytical solution is available only for the final regime, we can investigate the properties of the model only by solving it numerically. Section 2 of the Appendix discusses the numerical solution procedure.

Although the difficulties of solving such a dynamic system limit us to a very simplified and stylized representation of the global economy, we nevertheless want to investigate the solutions for as realistic a situation as possible. Section 3 of the Appendix outlines how we used data from a number of sources including the *Energy Information Administration* (EIA),[21] the *Survey of Energy Resources 2007* produced by the *World Energy Council*,[22] and *The GTAP 7 Data Base* produced by the *Center for Global Trade Analysis* in the Department of Agricultural Economics, Purdue University[23] to calibrate the parameter values and derive starting values for the endogenous eco-

20. Formally, if $v(\tau)<0$ for $\tau<T_1$, since $\partial g/\partial N<0$, (21) would imply $\dot{v}<0$ and $v<0$ for all $t>\tau$ contradicting $v(T_1)=0$.

21. International data is available at http://www.eia.doe.gov/emeu/international/contents.html

22. See http://www.worldenergy.org/publications/survey_of_energy_resources_2007/default.asp The data are estimates as of the end of 2005.

23. Information can be found at https://www.gtap.agecon.purdue.edu/databases/v7/default.asp The GTAP 7 data base pertains to data for 2004.

Table 1: Calibrated Parameter and Initial Variable Values

Parameter or Variable	Brief Description	Value
$Q(0)$	Initial population†	1.0
π	Population growth rate	0.01
β	Time discount rate	0.05
γ	Coefficient of relative risk aversion	4.0
$y(0)$	Initial per capita output†	1.0
$k(0)$	Initial per capita capital stock‡	3.6071
A	Input-output coefficient in production $y(0)/k(0)$	0.2772
δ	Depreciation rate for capital	0.04
$i(0)$	Initial per capita investment in capital k‡	0.219
$n(0)$	Initial per capita investment in fossil fuel technology N‡	0.0083
$c(0)$	Initial per capita consumption‡	0.662
$g(0,0)$	Initial per capita real marginal cost of fossil energy	0.1107
$R(0)$	Initial (fossil fuel only) energy input to production†	1.0
$\bar{S}$	Feasible technically recoverable fossil fuel resources§	2126.0527
$\bar{S}-\alpha_2/\alpha_3$	Initial producing reserves of fossil fuels§	15.361
α	Renewable energy knowledge productivity	0.25
ψ	Learning by doing for renewable technology knowledge	0.33
$1-\psi$	R&D effect on renewable technology knowledge	0.67
$p(0)=\Gamma_1^{-\alpha}$	Initial per capita real marginal cost of renewable energy	0.4428
Γ_2	Final per capita real marginal cost of renewable energy	0.08856

† Set equal to 1.0 by using the value at $t=0$ to define units
‡ Measured in units of $y(0)$
§ Measured in units of $R(0)$

nomic and energy system variables. Table 1 summarizes the values chosen for the parameters and initial values of endogenous variables.

4. RESULTS

The transition to renewable energy occurs after $T_1 = 88.41$ years. It then takes a little more than 227 years (until $T_2 = 315.8$) for H to attain its maximum value. Direct R&D expenditure j is then no longer worthwhile. World output per capita grows at an average annual rate of 4.22% in the fossil regime, 3.11% in the renewable regime with investment in R&D, and 4.07% in the long run with renewable energy at its minimum cost.[24]

Figure 3 shows the behavior of the main variables in the economy during the fossil fuel regimes.[25] The period over which $n = 0$ is very short, lasting just 0.0982 of a year. Once investment

24. The economic growth rates produced by the model are too large. For example, for the sample of countries in the Penn World Tables, the average annual growth rate of real GDP per capita over the period 2004–2011 was slightly over 2.63%. Similarly, for the sample of countries reported in International Energy Agency (IEA) statistics, the average annual growth in real GDP per capita over the period 2004–2012 was 2.7%. The global financial crisis of 2007–08 no doubt reduced these rates, and our model does not allow for business cycle fluctuations. Nevertheless, economic growth rates should be smaller to better fit the evidence. Equation (28) implies that the long run growth rate $-\bar{A}/\gamma$ declines as γ increases. We found, however, that higher values for γ, and thus lower growth rates, extend the time horizon of the model, increase roundoff errors, and make the model impossible to solve. Smaller values of γ produced higher economic growth rates, but had no significantly different effects on the key "energy crisis" aspect of the results.

25. We also can compare the results graphed in Figure 4 with evidence. With A constant, the model will give the same growth in k and y. Comparing GTAP data for 2004 and 2007, the ratio of y to k increased from 0.2772 to 0.3119. As we noted above, however, the "capital stock" in our model should be thought of as an amalgam of physical and human capital

Figure 3: Fossil Fuel Regime

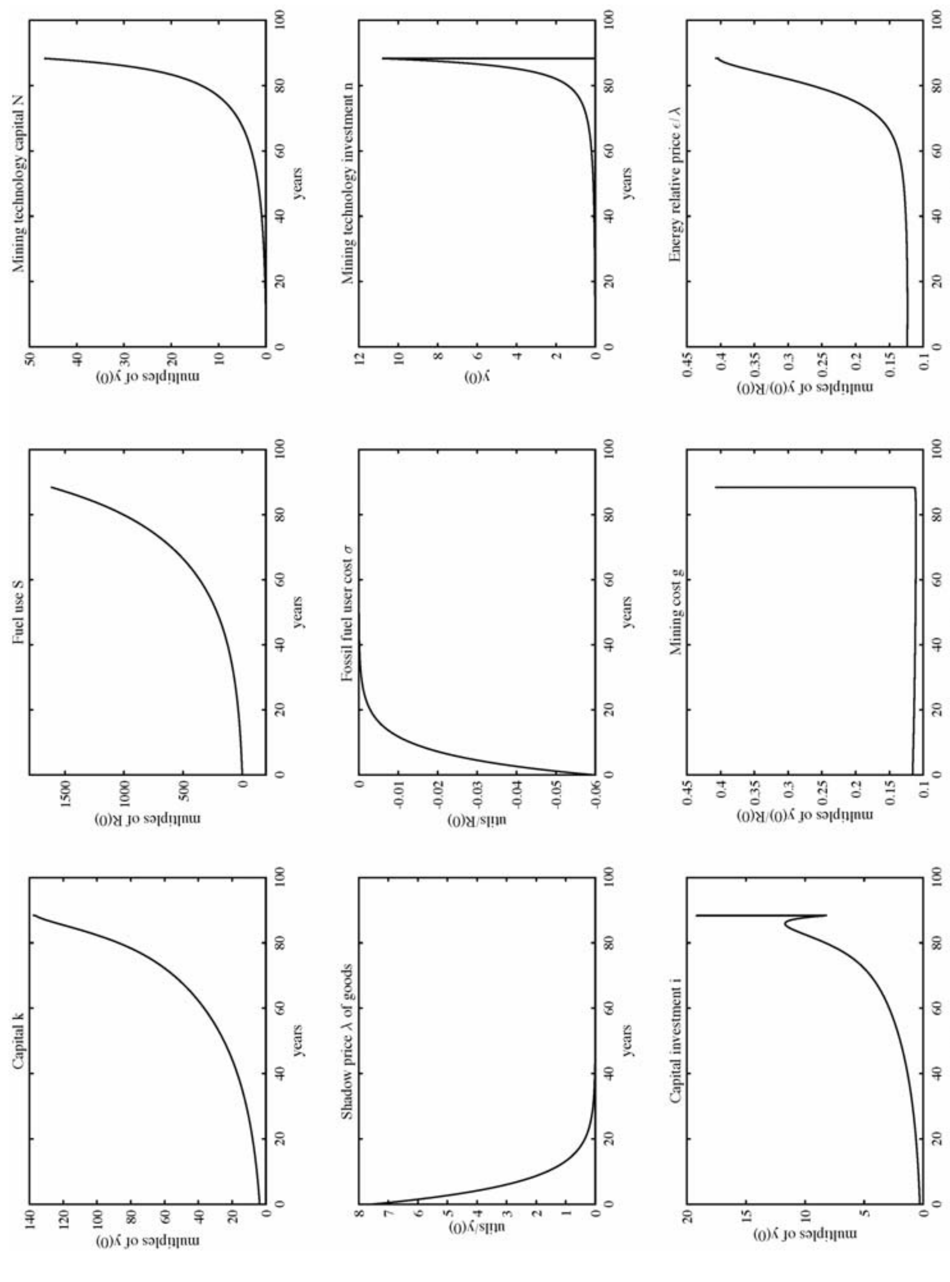
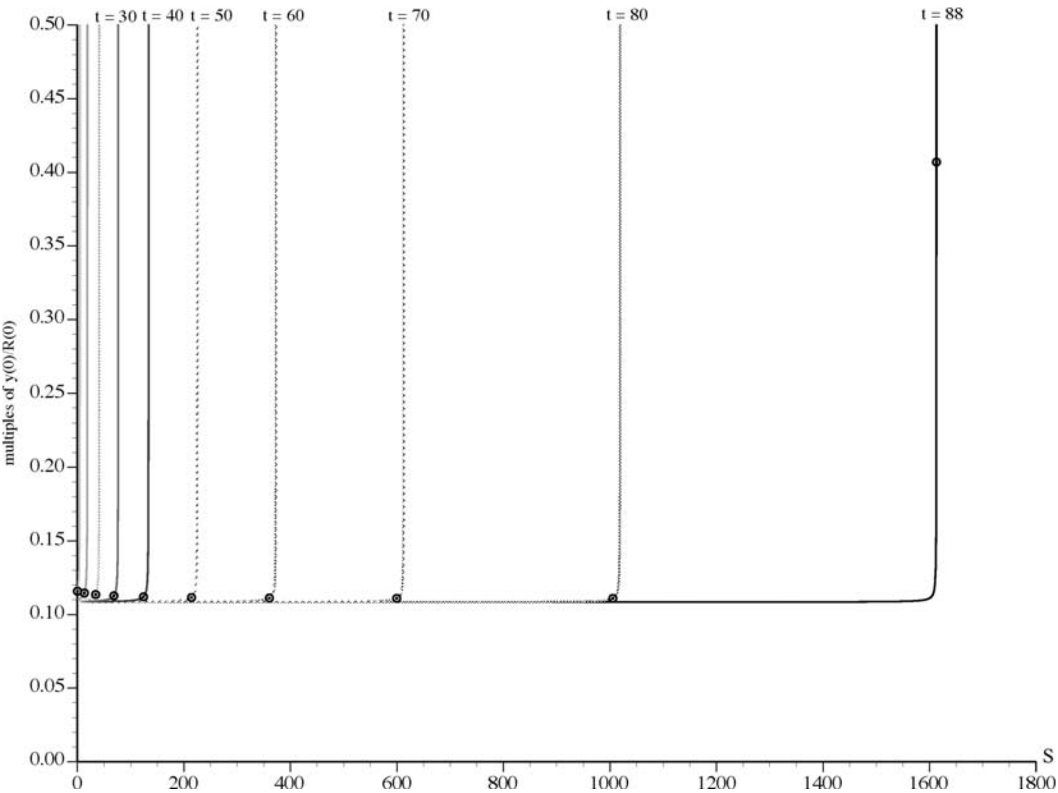


Figure 4: Selected Fossil Energy Cost Functions



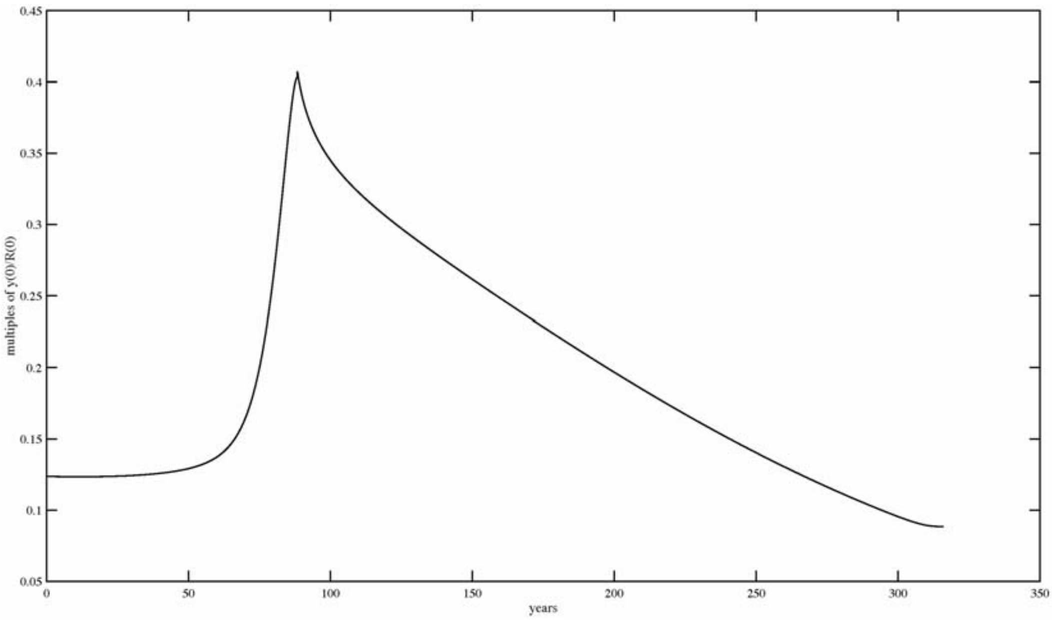
n ceases, the cost of fossil fuel rises dramatically and the transition to renewables follows soon thereafter. Prior to its plunge to zero, however, n rises dramatically as increasing amounts of investment are needed to offset the effects of depletion and maintain g roughly constant. The rise in n in turn constrains c and i, slowing the accumulation of k.

The explicit cost of fossil energy services $g(S,N)$ stays fairly constant during the fossil fuel regime as investment in N offsets the effect of increased S. This is shown in more detail in Figure 4, which plots $g(S,N)$ as a function of S for several years. The circled points give the actual costs as determined by the relevant value of S for each year.

Figure 5 illustrates that the "cost parity target" for renewables is a moving one. Technological change in the production and use of fossil fuel energy allows it to remain competitive for

with i covering investment in both. Using the GTAP data from 2007 and 2004 thus would understate the accumulation of k as specified by the model, and thus overstate the increase in y/k. The other critical variable is fossil fuel consumption. The model assumes a constant ratio of energy services input to output. However, data from the EIA records an average decline of 1.2% per annum in the ratio of global primary energy consumption to global real GDP over the period 2004–2011. There was an even larger average annual decline of 1.93% over the period 2004–2012 for the countries in the IEA data set. In addition, the model overstates economic growth. Hence, it would considerably overstate fossil fuel consumption. The critical factor in determining the transition to renewable energy, however, is not fossil fuel consumption per se but the rise in the cost of fossil fuel services. The increase of N by more than 0.0755 in the first 8 years of the model allows $g(S,N)$ to decline slightly despite the growth in S.

Figure 5: Relative Price of Energy



longer. Ultimately, the model implies about 80% of the technically recoverable fossil fuel resources are exploited, with the transition occurring in the last decade of this century.

Although the explicit cost of fossil energy supply slightly declines during most of the fossil fuel regime, Figure 5 shows that the shadow relative price of energy (ϵ/λ) rises continuously. The gap results from the rising user cost, or scarcity rent, of fossil fuels. Until σ/λ jumps to zero when fossil fuel use ceases, it becomes more negative over time.

Figure 6 shows the behavior of the main variables while renewable energy undergoes technological progress. After a brief initial "burst" of investment in R&D right after the transition, which steeply cuts the cost of renewable energy, direct investment in renewable R&D then drops close to zero. It subsequently gradually increases over time before plunging toward zero as the technological frontier for renewable energy efficiency looms. Evidently, for much of the "middle period" of this regime, learning by doing is a major source for accummulating technical knowledge.

Figure 7 focuses on the central issue of growth in per capita output and consumption.[26] Per capita consumption grows by an average 3.68% in the fossil energy regime, which is less than average output growth. By contrast, in the renewable regime with R&D, the declining cost of energy allows consumption growth at 3.33% to exceed output growth of 3.11%.

As concave utility should imply, consumption grows somewhat more smoothly than output. In particular, Figure 7 shows that the per capita output growth rate rises substantially for some time before the switch point T_1, and then plummets to be slightly negative right around T_1. Per capita consumption growth declines somewhat in the lead up to T_1, but stays above per capita output

26. In the long run regime, per capita output, consumption, investment in capital, and energy use all grow at the same average annual rate.

Figure 6: Renewable Regime

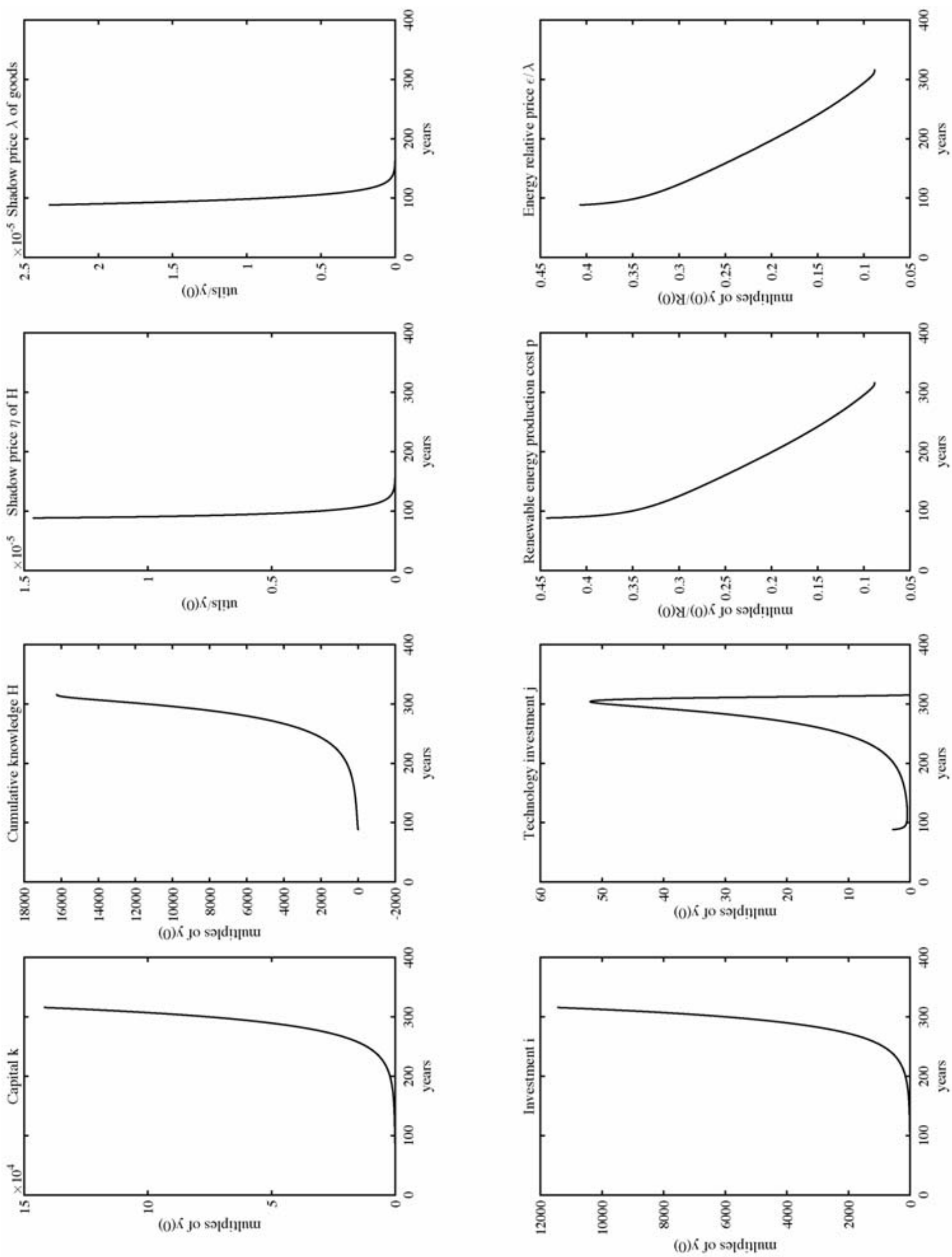
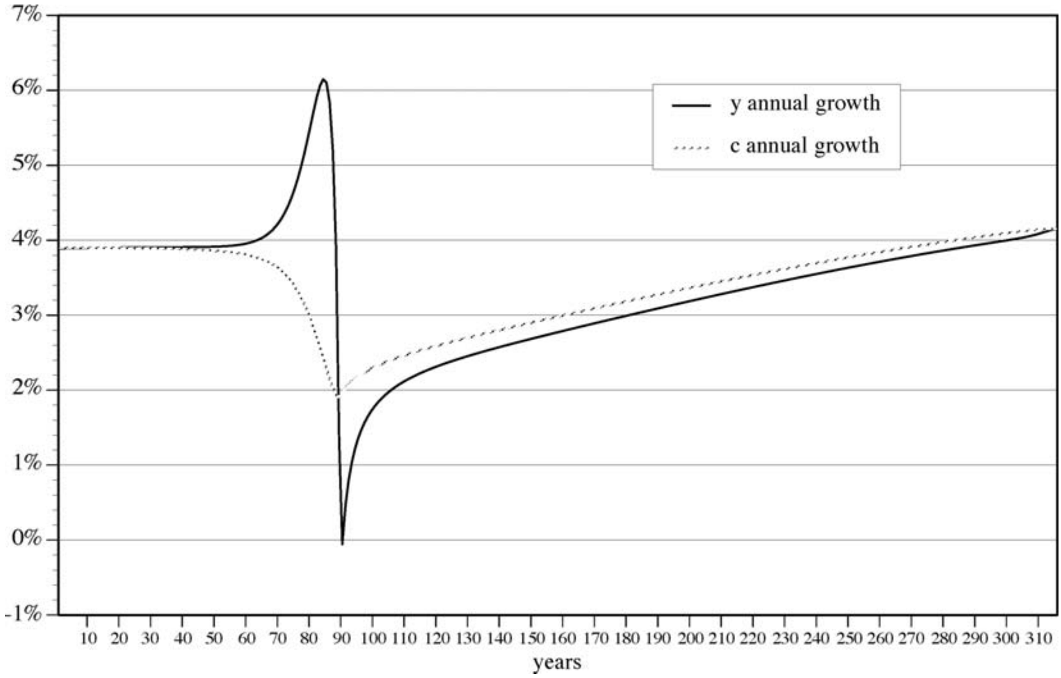


Figure 7: Annual Growth Rates of Per Capita Output and Consumption



growth after T_1. The result of both trends is that the share of consumption in output declines substantially around the time of the transition to renewables as illustrated in Figure 8.

Towards the end of the fossil fuel regime, the relative price of energy rises substantially. In addition, investments n in fossil technology are large in real terms. Hence, in the lead up to T_1 all of the additional output and more is absorbed into producing, and investing in, fossil energy leaving fewer resources for consumption. In summary, our model predicts an "energy crisis" around the switch point.

The consumption share, and the growth in per capita output and consumption, take a long time to recover to fossil fuel era levels after T_1. The explanation is that the cost of energy remains above the initial cost of fossil fuels for a considerable period of time. This is apparent in Figure 5, which shows that the shadow relative price of energy remains more than double the current level for over 75 years around the switch time (ten years before, 65 years after the switch time).

As we showed algebraically, the explicit cost of fossil energy is below the cost of renewable energy when the transition occurs. The learning by doing element of renewable energy production lowers the"full cost" of renewable energy, making it worthwhile to transition before the explicit cost of fossil energy reaches the initial cost of renewable energy.

The sharp fluctuations in investment in n and j noticed in Figures 3 and 6 come at the expense of similar sharp fluctuations in investment i in k. This is illustrated in Figures 9 and 10, which show different investment shares in output. In particular, Figure 10 shows that the sums of investment in k and energy technologies are much smoother than any of the investments taken alone.

Figure 8: Consumption Share of Output

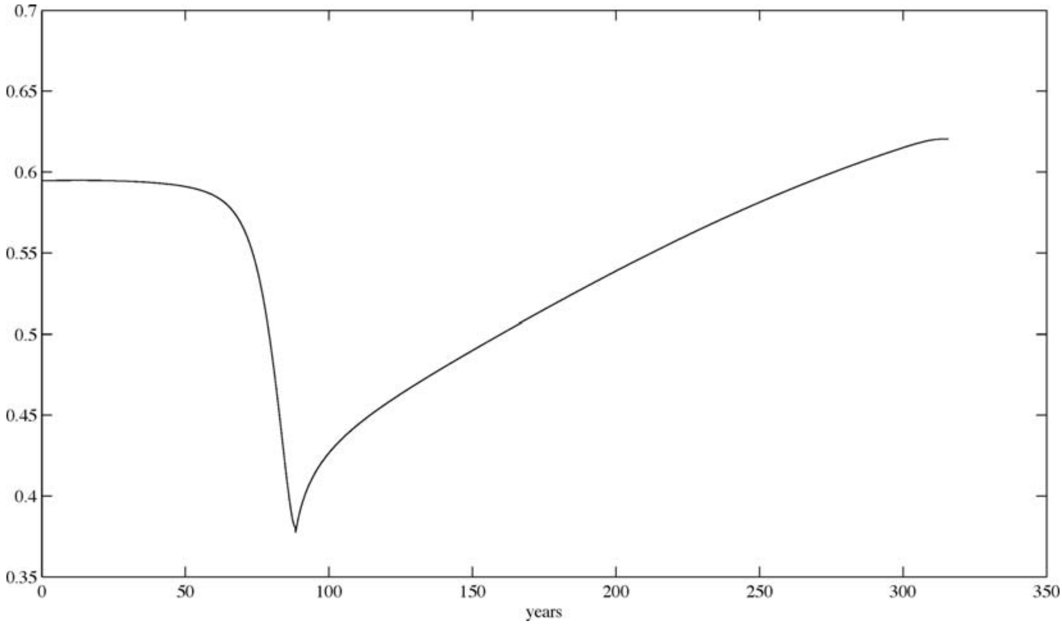
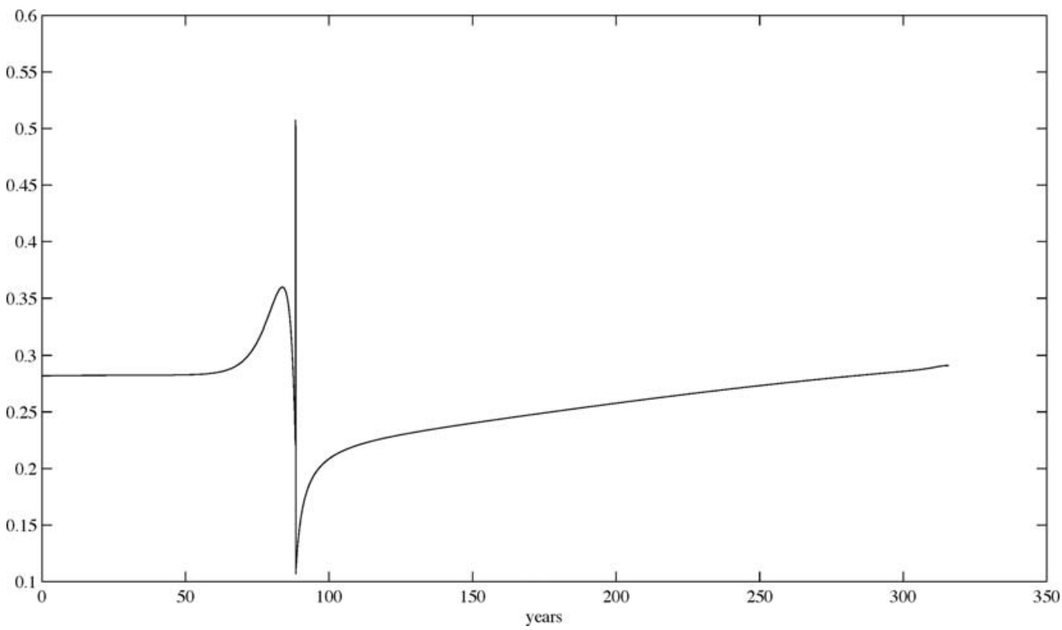


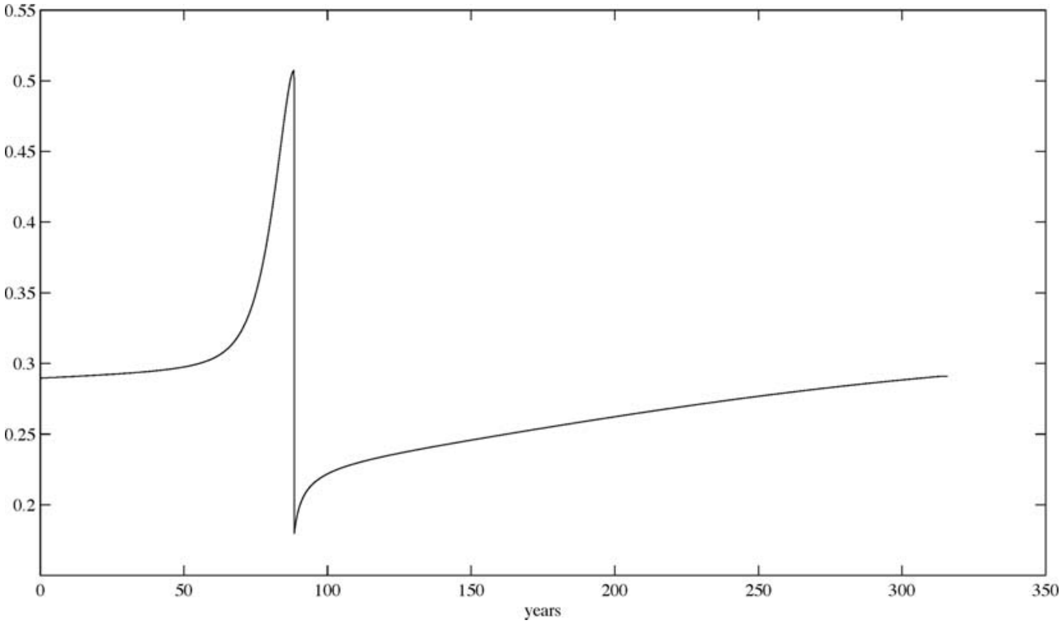
Figure 9: Output Share of Investment *i*



5. CONCLUSION

We studied the optimal transition from fossil fuel to renewable energy sources in a neoclassical growth economy with endogenous investment in new technology in both energy sectors.

Figure 10: Output Share of $i + n$ and $i + j$



After calibrating the model using data on the world economy in 2004, we found that the model predicts the transition to renewable energy will occur toward the end of this century when about 80% of the available fossil fuels will have been exploited. Innovations in technology keep the cost of fossil fuel energy services fairly constant even as increased exploitation raises mining costs. Thus, renewable technologies face a moving "parity target." Nevertheless, anticipation of the benefits of learning by doing imply that it is optimal to shift from fossil to renewable energy sources *before* fossil fuel costs rise to match the cost of renewables.

The share of consumption in output and the growth rate in per capita consumption both decline for several decades before the switch. Immediately around the switch point, per capita output growth becomes negative, but for some time prior to the switch per capita output growth increases. The consumption and output growth rates diverge because the rising cost of energy and increased investment in fossil energy technology absorb more than the output increase. A large investment in fossil fuel technology toward the end of the fossil regime is needed to offset the effects of depletion on energy costs. The shadow price of energy peaks at the end of the fossil fuel regime and remains more than double current levels for over 75 years around the switch time. After the switch, the high cost of energy and the need for continuing investment in improving renewable energy technologies continue to constrain the growth in per capita consumption and output for an extremely long time. Thus, our model predicts an "energy crisis" around the switch point and continuing slow growth for some time thereafter. This crisis is part of the efficient arrangement in our economy.

It is worth discussing the robustness of our findings. Although the two-factor learning model is standard in the literature, it requires both R&D and learning by doing for cost reductions to take place. Hence, the model does not allow the use of fossil fuel to coexist with progress in renewable technologies. If R&D investment alone could reduce costs of deploying renewable technologies, the cost of renewable technologies at the transition point could be lower. This would hasten the transition and, since the increase in the relative price of energy would be moderated, the energy crisis would be less severe. However, there would still be an energy price maximum during the transition, and the increased investment in fossil fuels (to offset the effects of depletion) would remain. Thus, we believe that our results would not change qualitatively. At the least, our model suggests a novel connection between the magnitude of the energy crisis at the transition point and the substitutability between innovation and learning by doing effects in renewable energy production.

Our model also assumes perfect substitutability between alternative energy sources. Using a continuity argument, we can show that our results remain true if the degree of substitutability is high, but not perfect. We have also examined a model with energy-source specific capital and investments in end-use energy efficiency. Although the additional state variables make such a model more difficult to solve, preliminary results show that the "energy crisis" around the transition point between fossil and non-fossil energy sources remains.

We have also studied decentralized allocations in a discrete time analog of the model. This allows us to explicitly account for externalities associated with the investment process and the possibility of under-investment in R&D. Such deviations from efficiency allow a possible role for policy. The higher relative price of energy, and higher levels of investment in energy technologies around the transition point, nevertheless imply that the "energy crisis" remains a feature of such extended models.

Finally, we reiterate that our analysis abstracts from two important factors relevant to policy: energy independence and the environmental costs from fossil fuel combustion. While environmental factors will likely favor renewables, incorporating benefits from energy independence could favor both renewable and (unconventional) fossil fuel sources. Investigating these issues is another important topic for future research.

6. APPENDIX

6.1 Energy Regimes

We begin our analysis with the last regime and then proceed backwards through time.

6.1.1 The Long Run Endogenous Growth Economy

With p constant at Γ_2, we obtain a simple endogenous growth model with investment only in k. We retain first order conditions (12), (13) and (17), co-state equation (18), the resource constraint (10) and differential equation (2). However, (17) changes to $\epsilon = \lambda\Gamma_2$. From (13) we obtain $q = \lambda$ and hence $\dot{q} = \dot{\lambda}$, and co-state equation (18) becomes

$$\dot{\lambda} = [\beta + \delta - (1-\Gamma_2)A]\,\lambda \equiv \bar{A}\lambda \tag{24}$$

where $\bar{A}$ is constant. For perpetual growth, we need $c \to \infty$ as $t \to \infty$, which from (12) requires $\lambda \to 0$ and hence $\bar{A} < 0$, that is,[27] $A(1-\Gamma_2) > \beta + \delta$. The solution to (24) is

$$\lambda = \bar{K}e^{\bar{A}t} \tag{25}$$

27. Thus, output per unit of capital *net* of renewable energy costs must exceed depreciation plus the time discount factor.

for some constant $\bar{K}$. Then the resource constraint, (12) and (25) imply

$$\dot{k} = (\beta - \bar{A})k - \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma} \tag{26}$$

which, for another constant C_0, has the solution

$$k = C_0 e^{(\beta - \bar{A})t} + \frac{\gamma \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma}}{\beta\gamma - \bar{A}(\gamma - 1)} \tag{27}$$

However, the transversality condition at infinity, $\lim_{t\to\infty} e^{-\beta t}\lambda k = 0$, requires $C_0 = 0$ and $\bar{A}(\gamma - 1) < \beta\gamma$.[28] In summary, k in the final endogenous growth economy will be given by

$$k = \frac{\gamma \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma}}{\beta\gamma - \bar{A}(\gamma - 1)} \tag{28}$$

with λ given by (25) and $\bar{K}$ is a constant yet to be determined.

6.1.2 Renewables with Technological Progress

When $B = Ak > 0$, $j > 0$ and $H < \Gamma_2^{-1/\alpha} - \Gamma_1$, j is given by (22). Hence, $\dot{H}$ will satisfy:[29]

$$\dot{H} = [(1-\psi)(\eta/\lambda)]^{(1-\psi)/\psi} B = [(1-\psi)(\eta/\lambda)]^{(1-\psi)/\psi} Ak \tag{29}$$

For $B > 0$, (17) implies $\zeta = 0$, while $H < \Gamma_2^{-1/\alpha} - \Gamma_1$ and (19) imply $\chi = 0$. The solution (22) for j therefore also implies that the shadow price of energy will be given by:

$$\epsilon = \lambda(\Gamma_1 + H)^{-\alpha} - \psi(1-\psi)^{(1-\psi)/\psi} \lambda^{(\psi-1)/\psi} \eta^{1/\psi} \tag{30}$$

Substituting (30) into (18) and noting that $q = \lambda$ implies $\dot{q} = \dot{\lambda}$, we obtain

$$\dot{\lambda} = [\beta + \delta - A(1 - (\Gamma_1 + H)^{-\alpha})]\lambda - \psi A(1-\psi)^{(1-\psi)/\psi} \lambda^{(\psi-1)/\psi} \eta^{1/\psi} \tag{31}$$

From (19) with $B = Ak$, we obtain

$$\dot{\eta} = \beta\eta - \lambda\alpha(\Gamma_1 + H)^{-\alpha-1} Ak \tag{32}$$

The resource constraint, the first order condition (12) for c and the solution (22) for j with $B = Ak$ then determine i and hence the differential equation for $\dot{k}$:

$$\dot{k} = Ak[1 - (\Gamma_1 + H)^{-\alpha} - (1-\psi)^{1/\psi} \eta^{1/\psi} \lambda^{-1/\psi}] - \lambda^{-1/\gamma} - \delta k \tag{33}$$

28. Note that since $\bar{A} < 0$ the inequality will be satisfied if $\gamma > 1$, while if $0 < \gamma < 1$, it will require $\Gamma_2 > 1 - [\beta/(1-\gamma) + \delta]/A$. Thus, for $\gamma < 1$, we need $\beta/(1-\gamma) > A(1-\Gamma_2) - \delta > \beta$.

29. If η/λ evolves slowly, H will approximately equal a constant times accumulated production B. Since (22) implies that direct R&D grows with B, empirical studies could find a power law relationship between energy production cost and accumulated output *alone* even though explicit R&D is more important in reducing costs.

This regime therefore is characterized by four simultaneous differential equations (29), (31), (32) and (33) for the four state and co-state variables k, H, η, and λ.

6.1.3 The Initial Fossil Fuel Economy

In the initial regime where $R>0$, (16) implies $\xi=0$ and the shadow price of energy is[30]

$$\epsilon = \lambda g(S,N) - \sigma Q \tag{34}$$

While $n>0$, (15) implies $\omega=0$ and hence $v=\lambda$. But then $\dot{v}=\dot{\lambda}$ and (21) implies

$$\dot{\lambda} = \beta\lambda + \lambda \frac{\partial g}{\partial N} R \tag{35}$$

If we also have $i>0$, (13) will imply $\lambda = q$ and from (18) and (34), we will also have $\dot{\lambda} = (\beta + \delta + g(S,N)A - A)\lambda - \sigma QA$. Using (35) we then conclude

$$\left[\delta + g(S,N)A - \frac{\partial g}{\partial N}R - A\right]\lambda = \sigma QA \tag{36}$$

Since $\sigma<0$ and $\lambda = c^{-\gamma}>0$, a necessary condition for (36) to hold is that

$$\delta + g(S,N)A - \frac{\partial g}{\partial N}R < A \tag{37}$$

Condition (37) must eventually be violated because increasing S must raise $g(S,N)$ above 1 and $\partial g/\partial N<0$. Thus, we cannot have $R>0$ and $n>0$ forever, but we assume that $R>0$ and $n>0$ at $t=0$.

Substituting $R=Ak$ into (36), we obtain an equation relating N and k. After differentiating with respect to time, substituting for $\dot{N}, \dot{\lambda}/\lambda = \dot{v}/v, \dot{S}, \dot{\sigma}$ and $\dot{Q} = \pi Q$ (since the exogenous growth rate of Q is π), and using (36), we obtain a relationship between i and n:

$$\lambda\left[\frac{\partial g}{\partial N}\left(n + \delta k + \frac{\sigma QAk}{\lambda} - i\right) - \frac{\partial^2 g}{\partial S \partial N} QAk^2 - \frac{\partial^2 g}{\partial N^2} nk\right] = \sigma\pi Q \tag{38}$$

Using the result $j=0$ if $B=0$, (12), (3) and (4), the resource constraint (10) implies:

$$i = Ak[1 - g(S,N)] - \lambda^{-1/\gamma} - n \tag{39}$$

Substituting (39) into (38), we then obtain an equation to be solved for n:

$$\begin{aligned} n\lambda\left(\frac{\partial^2 g}{\partial N^2}k - 2\frac{\partial g}{\partial N}\right) &= -\sigma\pi Q + \\ \lambda\left[\frac{\partial g}{\partial N}\left[k\left(\delta + g(S,N)A - A + \frac{\sigma QA}{\lambda}\right) + \lambda^{-1/\gamma}\right] - \frac{\partial^2 g}{\partial S \partial N} QAk^2\right] & \end{aligned} \tag{40}$$

30. As noted in Section 3.4, $\sigma<0$ until T_1 when it jumps to zero, so (34) implies $\epsilon>0$.

Using the signs of the partial derivatives of g, one can show that (40) likely yields $n>0$ as hypothesized.[31] Then (39) can be solved for i.

In summary, the initial period of fossil fuel use produces five simultaneous differential equations (2), (6), $\dot{S} = QAk$, $\dot{\lambda} = \lambda[\beta + \delta + (g(S,N) - 1)A] - \sigma QA$ and $\dot{\sigma} = \beta\sigma + \lambda Ak(\partial g/\partial S)$ for k, N, S, λ, and σ together with the exogenous population growth $Q = Q_0 e^{\pi t}$.

As we argued in Section 3.4, the region where $R>0$ and $n>0$ will end at some $T_0 < T_1$ and between T_0 and T_1, we will have $n = 0$ and $R>0$. In this region, N is fixed at $\bar{N}$, and the resource constraint together with the first order condition (12) for c will imply

$$i = Ak[1 - g(S,\bar{N})] - \lambda^{-1/\gamma} \tag{41}$$

In this region, $\dot{v}$ will be given by (21) and v will evolve separately from λ. The equations for $\dot{k}, \dot{S}$, $\dot{\lambda}$ and $\dot{\sigma}$ will continue to be the same as in the fossil fuel regime with $n>0$.

6.2 The Numerical Solution Procedure

Mirroring the theoretical analysis, it is easier to solve the model backwards through time. The known initial values $S(0) = N(0) = 0$, $k(0) = k_0 > 0$ of the state variables at $t = 0$ then become targets. We have to set three free variables in order to hit these three target values. If we guess values for T_2 and the capital stock at that time $k(T_2)$, the values of $\bar{K}$ and hence $\lambda(T_2)$ are also determined. We also know that at T_2 we must have $\eta(T_2) = 0$ and $p = (\Gamma_1 + H)^{-\alpha} = \Gamma_2$, which will determine the value of H at T_2, namely $H = \Gamma_2^{-1/\alpha} - \Gamma_1$. The differential equations (29), (31), (32) and (33) are then solved backward until T_1, when $H = 0$. The values of k and λ at T_1 then provide initial conditions for the differential equations for $\dot{k}$ and $\dot{\lambda}$ in the fossil fuel regime. Using (30) and (34), the fact that $\sigma(T_1) = 0$, and the requirement that the shadow price of energy has to be continuous we conclude that

$$\Gamma_1^{-\alpha} - \frac{\eta}{\lambda} = \frac{\epsilon}{\lambda} = g(S,N) \tag{42}$$

For the values of $\eta(T_1)$ and $\lambda(T_1)$ obtained from the backward solution in the renewable regime, and the exogenously specified $\Gamma_1^{-\alpha}$, (42) would then determine the value of $g(S(T_1),N(T_1))$ at T_1. Thus, $N(T_1)$ will be determined once we guess the value of $S(T_1)$. Finally, the requirements that $\sigma(T_1) = 0 = v(T_1)$ will provide the remaining initial conditions for the five differential equations for k, S, λ, v and σ in the fossil fuel regime with $n = 0$. The initial fossil fuel regime with $n>0$ then starts at T_0 when $v = \lambda$. For all $t \le T_0$, we then use five simultaneous differential equations to solve for k, S, N, σ and λ.

As noted in the next section, we also targeted initial values for n and c by freeing up two

31. Since $\partial g/\partial N < 0$ and $\partial^2 g/\partial N^2 > 0$, the coefficient of n on the left hand side of (40) is positive. From the resource constraint (39), $\delta k + Ak(g-1) + \lambda^{-1/\gamma} = \delta k - i - n \le \delta k - n$. Thus $n>0$ if

$$-\frac{\partial^2 g}{\partial S \partial N} QAk^2 + \frac{\partial g}{\partial N}\left(\delta + \frac{\sigma QA}{\lambda}\right)k - \sigma\pi Q > 0$$

Since $\partial^2 g/\partial S \partial N < 0$ and $\sigma < 0$, the quadratic in k has a positive second derivative and positive intercept, so even if $\delta + \sigma QA/\lambda > 0$ we conclude that the expression must be positive for large k. For small values of k, we are likely to have $\dot{k} = i - \delta k > 0$, in which case the right hand side of (40) is guaranteed to be positive.

additional parameter values, α_3 and $g_S(0)$. To solve the model, we guessed values for α_3 and $g_S(0)$ and then solved the differential equations backwards in MatLab for many different values of $T_2, k(T_2)$ and $S(T_1)$ in an attempt to attain $k(0) = 3.6071, N(0) = 0 = S(0), n(0) = 0.0083$ and $c(0) = 0.6620$. We then adjusted α_3 and $g_S(0)$ and repeated the procedure. The closest we could get[32] resulted from setting $\alpha_3 = 15, g_S(0) = 0.00015, T_2 = 315.8, k(T_2) = 141704.98998437249$ and $S(T_1) = 1613$, which yielded calculated initial values of $k(0) = 3.6644, S(0) = -0.0003179340598, N(0) = -0.07501979386, n(0) = 0.0077708$ and $c(0) = 0.60402$. We also verified that a discrete time approximation to the continuous time model gave essentially the same solution for the same parameter values. It may also be worth noting that in carrying out this procedure we solved the differential equations for many pairs of values of α_3 and $g_S(0)$, and thus in effect a range of g functions. In every case, the solution paths looked very similar to the ones presented in the paper and, in particular, they all displayed a similar "energy crisis" around the transition point between energy regimes.

6.3 Calibrating the Model

As noted in the text, $Q(0)$, $y(0)$ and $R(0)$ were all set to 1.0 by an appropriate definition of units. We assume a world population growth rate π of 1%.[33] In line with standard assumptions made to calibrate growth models, we assume a continuous time discount rate $\beta = 0.05$ and a coefficient of relative risk aversion $\gamma = 4$.[34]

To calibrate initial values for the macroeconomic variables we first subtracted government spending from GDP since it does not appear in the model.[35] We then define units of output so that initial private sector expenditur $y(0)$ is 1. Converting the GTAP data base estimates of the world capital stock to units of $y(0)$, we obtain $k(0) = 3.6071$ and $A \approx 0.2772$. We also use the GTAP depreciation rate on capital of 4% for δ. From the GTAP investment share of private sector expenditure we obtain $i(0) + n(0) = 0.2273$ in units where $y(0)$ is defined to be 1. Using capital shares by sector, we estimate that around 3.66% of annual investment occurs in the oil, natural gas, coal, electricity, and gas distribution sectors.[36] Thus, we set $n(0) \approx 0.0083$ and $i(0) \approx 0.2190$. From the resource constraint (10), the difference between output and the sum of the investments, namely 0.7727 would equal $c(0) + gR$. Classifying the combined spending on the primary fuels coal, oil and natural gas and the energy commodity transformation sectors of refining, chemicals, electricity

32. The long time horizon resulted in calculations being close to the limit of numerical accuracy. For example, we needed to set the tolerance levels for the differential equation solvers to $5.0 * 10^{-14}$.

33. This is consistent with extrapolating recent world growth rates reported by the UN Food And Agriculture Organization, http://faostat.fao.org/site/550/default.aspx

34. There is no strong consensus on the latter, but it is usually taken to lie between one and ten. As noted in the text, we would prefer to use a larger value for γ to better match observed economic growth rates. However, increased round-off errors with even slightly larger values of γ prevented us from finding a solution.

35. Government spending would not affect the equilibrium if it was financed by lump sum taxes and the utility obtained from it was additively separable from the utility obtained from private consumption. In any case, since government spending is concentrated on services rather than production processes using energy as a significant input, we believe that including government spending would not significantly alter the results.

36. Since we have defined R to be energy services input, investments in energy efficiency also increase the effective supply of fossil fuels. In particular, investments in the energy transformation sectors are included in n to account for the fact that some of these expenditures would be directed toward increasing energy efficiency. While this would overstate investments in energy efficiency, some investments in the transportation and manufacturing sectors that have not been included in n would be aimed at raising energy efficiency.

generation and natural gas distribution as "energy expenditure" we obtain $gR = 0.1107$. Subtracting this from 0.7727 we obtain $c(0) = 0.6620$.[37]

Turning next to the initial values of the energy variables, we note that after defining the initial values of S and N to be zero, the initial value for gR would imply

$$\frac{0.1107}{R(0)} = \alpha_0 + \frac{\alpha_1}{\bar{S} - \alpha_2/\alpha_3} \tag{43}$$

We define units so initial fossil fuel production $R(0) = 1$, but to estimate total fossil fuel resources[38] $\bar{S}$ in the same units we need $R(0)$ in energy units. The EIA web site gives total global production of oil, natural gas and coal in 2004 as 392.689 quads. Using conversion factors also available at the EIA, World Energy Council estimates of additional resources in place measured in millions of tonnes of coal, millions of barrels of oil, extra heavy oil, natural bitumen and oil shale and trillions of cubic feet of natural gas were converted to 115.2 quintillion BTU, or almost 300 times $R(0)$. These resources are nevertheless small compared to estimates of the energy that may be recovered from methane hydrates. Perhaps because a commercially viable process for producing methane hydrates is yet to be demonstrated, resource estimates vary widely. According to the National Energy Technology Laboratory (NETL),[39] the United States Geological Survey (USGS) has estimated potential resources of about 200,000 trillion cubic feet in the United States alone. According to Timothy Collett of the USGS,[40] current estimates of the worldwide resource in place are about 700,000 trillion cubic feet of methane. Using the latter figure, this would be equivalent to 719.6 quintillion BTU. Adding this to the previous total of oil, natural gas and coal resources yields a value for $\bar{S} = 834.8$ quintillion BTU or around 2126.0527 times $R(0)$. Equation (43) with $R \equiv 1$ and $\bar{S} = 2126.0527$ then gives one equation in four unknowns. Recall that $\bar{S} - \alpha_2/\alpha_3$ is the level of fossil fuel extraction S at which marginal costs of extraction $g(S,0)$ become unbounded assuming no investment in N. We therefore associate $\bar{S} - \alpha_2/\alpha_3$ with total output available from existing fossil fuel producing reserves.[41] We calculate the producing reserves by inverting data on decline rates.[42] A recent Cambridge Energy Research Associates report (Jackson, 2009) notes that weighted average decline rates for existing oil fields is around 4.5%, but "the average decline rate for [oil] fields that were actually in the decline phase was . . . 6.1% when the numbers are production weighted." Hence, we shall use 6% as a decline rate for oil fields. Using United States production and reserve

37. The *model* solution for $c(0)$ would follow from the first order condition $\lambda(0) = c(0)^{-\gamma}$. To obtain a solution that matches the calibrated value for $c(0)$ we need to free up an additional parameter.

38. This should equal the feasible technically recoverable fossil fuel resources. The model determines endogenously how much of this resource base ends up being economically recoverable taking into account endogenous investment in new mining technology and changes in real energy prices. We thus took an expansive notion of what may be included in $\bar{S}$. As a referee pointed out, even what is considered "feasible technically recoverable" resource at any one time is not completely independent of current prices, so we also examined the effects of varying $\bar{S}$. Since the model is difficult and time consuming to solve, we restricted our investigation to small increases in $\bar{S}$. The only effect of these changes was an almost uniform delay in T_0, T_1 and T_2, and an increase in the total amount of fossil fuel exploited $S(T_1)$ that was only slightly above one-third the increase in $\bar{S}$. Thus, around two-thirds of the additional resource was "left in the ground."

39. http://www.netl.doe.gov/technologies/oil-gas/FutureSupply/MethaneHydrates/about-hydrates/estimates.htm

40. http://www.netl.doe.gov/kmd/cds/disk10/collett.pdf

41. Current *official* reserves are not the relevant measure since many of these are not currently being exploited, and thus are unavailable for production without further investment.

42. Unlike Venables (2014), we do not model physical decline in oil and gas wells. We are referring to decline rates only to obtain a measure of the current volume of producing reserves.

figures as a guide, natural gas decline rates are closer to 8% per year, but coal mine decline rates are closer to 6% per year. In accordance with these figures, we assume the ratio of "fossil fuel" production to producing reserves equals the share weighted average of these figures, namely $(175.948*0.06+100.141*0.08+116.6*0.06)/392.689=0.0651$. Hence, the initial value of producing reserves $\bar{S}-\alpha_2/\alpha_3$ can be written as $1/0.0651=15.361$ times $R(0)$. Using the previously calculated value for $\bar{S}$, this leads to $\alpha_2/\alpha_3=2110.538$. We can obtain two more equations by specifying α_3 and the partial derivative of g at $t=0$ in order to target two additional variables, $c(0)$ and $n(0)$.

We focus next on the learning curve (8). The literature provides a range of estimates for the effect α of learning or knowledge on costs. In a study of wind turbines, Coulomb and Neuhoff (2006) found values of α of 0.158 and 0.197. Grübler and Messner (1998) found a value of $\alpha=.36$ using data on solar panels. Bentham et al. (2008) report several studies finding a learning percentage of around 20% ($\alpha=0.322$) for solar panels. We conclude that for renewable energy technologies α could range from a low of 0.15 to a high of 0.32, so we chose a middle value of $\alpha=0.25$. To obtain a value for ψ, as noted in the text we use a study by Klaassen et. al. (2005) that estimated a two-factor learning model. Although they assume the capital cost is multiplicative in total R&D and cumulative capacity, while we assume the change in knowledge $\dot{H}$ is multiplicative in new R&D and current output, we take their parameter estimates as a guide. Since they find direct R&D is roughly twice as productive for reducing costs as is learning by doing, we assume that $\psi=0.33$.

Finally, we consider the initial cost $p(0)=\Gamma_1^{-\alpha}$, and long run cost Γ_2, of using renewable energy as the primary energy source. The EIA reports[43] that the capital cost of new onshore wind capacity is about double the cost of combined cycle gas turbines (CCGT), while offshore wind is around four times as expensive, solar thermal more than five times as expensive and solar photovoltaic more than six times as expensive. However, these costs do not take account of the lower average capacity factor of intermittent sources such as wind or solar. The same document gives a fixed O&M cost of onshore wind that is around two and a half times the corresponding fixed O&M for CCGT, although the latter also has fuel costs. The corresponding ratio is around seven for offshore wind, while fixed O&M for solar photovoltaic are similar to the fixed O&M for CCGT. As a rough approximation, we will assume $\Gamma_1^{-\alpha}$ is around four times the initial value of g. Following the EIA, we also assume that the renewable technologies can ultimately experience a five-fold reduction in costs, so $\Gamma_2=\Gamma_1^{-\alpha}/5$.

ACKNOWLEDGMENTS

We thank two anonymous referees and audiences at the Baker Institute Energy Forum, the Banco de Portugal, the Banque de France, CEMFI (Madrid, Spain), the CESifo Area Conference on Energy and Climate Economics, the European University Institute, the International Association for Energy Economics meetings in Rio and Stockholm, the Southern Methodist University, Texas A&M University, the Tsinghua Workshop in Macroeconomics, the University of Munich, the University of Queensland and the University of Western Australia for comments and suggestions.

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43. *Assumptions to the Annual Energy Outlook, 2009*, " Electricity Market Module," Table 8.2, available at http://www.eia.doe.gov/oiaf/aeo/assumption/pdf/electricity.pdf #page = 3

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