# Comparison of Congestion Management Techniques: Nodal, Zonal and Discriminatory Pricing

*Pär Holmberg\* and Ewa Lazarczyk\*\**

### ABSTRACT

Wholesale electricity markets use different market designs to handle congestion in the transmission network. We compare nodal, zonal and discriminatory pricing in general networks with transmission constraints and loop flows. We conclude that in large games with many producers and certain information, the three market designs result in the same efficient dispatch. However, zonal pricing with counter-trading results in additional payments to producers in export-constrained nodes, which leads to inefficient investments in the long-run.

**Keywords:** Congestion management, Wholesale electricity market, Transmission network, Nodal pricing, Zonal pricing, Counter-trading, Discriminatory pricing, Large game

## 1. INTRODUCTION

Storage possibilities are negligible in most electric power networks, so demand and supply must be instantly balanced. One consequence is that transmission constraints and the way they are managed can have a large influence on market prices. The European Union's regulation 1228/2003 (amended in 2006) sets out guidelines for how congestion should be managed in Europe. System operators should coordinate their decisions and choose designs that are secure, efficient, transparent and market based.

In this paper, we compare the efficiency and welfare distribution of three market designs that are in operation in real-time electricity markets: nodal, zonal and discriminatory pricing. Characteristics of the three designs are summarized in Table 1. The zonal market is special in that it has two stages: a zonal clearing and a redispatch. We show that in competitive markets without uncertainties the three designs result in the same efficient dispatch. However, zonal pricing with a market based redispatch (counter-trading) results in additional payments to producers in export-constrained nodes, as they can make an arbitrage profit from price differences between the zonal market and the redispatch stage. This strategy is often referred to as the *increase-decrease* (inc-dec) game. This is the first paper that proves these results for general networks with general production costs. Dijk and Willems (2011) are closest to our study. However, their analysis is limited to two-node networks and linear production costs. The parallel study by Ruderer and Zöttl (2012) is also analyzing similar

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\*      Corresponding author. Research Institute of Industrial Economics (IFN). P.O. Box 55665, SE-102 15 Stockholm, Sweden, phone +46 8 665 45 59, fax + 46 8 665 4599. E-mail: Par.Holmberg@ifn.se. Associate Researcher of Electricity Policy Research Group (EPRG), University of Cambridge.

\*\*    Research Institute of Industrial Economics (IFN). P.O. Box 55665, SE-102 15 Stockholm, Sweden, fax + 46 8 665 4599, Stockholm School of Economics, Department of Economics, Sveavägen 65, Stockholm, Sweden. E-mail: ewa.lazarczyk@hhs.se.

**Table 1:  Summary of the Three Congestion Management Techniques**

| Congestion management technique | Considered transmission constraints | Auction format Uniform-price | Auction format Pay-as-bid |
|---|---|---|---|
| Nodal | All | X | |
| Discriminatory | All | | X |
| Zonal–stage 1 | Inter-zonal | X | |
| Redispatch–stage 2 | Intra-zonal | | X |

issues, but the redispatch of the zonal market that they consider is not market based, thus their model does not capture the increase-decrease game.

**1.1 Congestion Management Techniques**

Producers submit offers to real-time markets just before electricity is going to be produced and delivered to consumers. During the delivery period, the system operator accepts offers in order to clear the real-time market, taking transmission constraints into account. The auction design decides upon accepted offers and their payments. Nodal pricing or locational marginal pricing (LMP) acknowledges that location is an important aspect of electricity which should be reflected in its price, so all accepted offers are paid a local uniform-price associated with each node of the electricity network (Schweppe et al., 1988; Hogan, 1992; Chao and Peck, 1996; Hsu, 1997). This design is used in Argentina, Chile, New Zealand, Russia, Singapore and in several U.S. states, e.g. Southwest Power Pool (SPP), California, New England, New York, PJM[1] and Texas. Nodal pricing is not yet used inside the European Union. However, Poland has serious discussions about implementing this design.

Under discriminatory pricing, where accepted offers are paid as bid, there is no uniform market price. Still, the system operator considers all transmission constraints when accepting offers, so there is locational pricing in the sense that production in import-constrained nodes can bid higher than production in export constrained nodes and still be accepted. Discriminatory pricing is used in Iran, in the British real-time market, and Italy has decided to implement it as well. A consequence of the pay-as-bid format is that accepted production is paid its stated production cost. Thus one (somewhat naïve) motivation for this auction format is that if producers would bid their true cost, then this format would increase consumers' and/or the auctioneer's welfare at producers' expense.

The third type of congestion management is zonal pricing. Markets, which use this design, consider inter-zonal congestion, but have a uniform market price inside each region, typically a country (continental Europe) or a state (Australia), regardless of transmission congestion inside the region. Denmark, Norway and Sweden[2] are also divided into several zones, but this division is motivated by properties of the network rather than by borders of administrative regions.[3] Britain is

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1. PJM is the largest deregulated wholesale electricity market, covering all or parts of 13 U.S. states and the District of Columbia.

2. The Swedish government introduced four zones in Sweden from November 2011, as a result of an antitrust settlement between the European commission and the Swedish network operator (Sadowska and Willems, 2012).

3. The optimal definition of zones for a given network is studied by e.g. Stoft (1997), Bjørndal and Jörnsten (2001) and Ehrenmann and Smeers (2005).

one zone in its day-ahead market, but uses discriminatory pricing in the real-time market. Initially the zonal design was thought to minimize the complexity of the pricing settlement and politically it is sometimes more acceptable to have just one price in a country/state.[4] Originally, zonal pricing was also used in the deregulated electricity markets of the U.S., but they have now switched to nodal pricing, at least for generation. One reason for this change in the U.S. is that zonal pricing is, contrary to its purpose, actually quite complex and the pricing system is not very transparent under the hood. The main problem with the zonal design is that after the zones of the real-time market have been cleared the system operator needs to order redispatches if transmission lines inside a zone would otherwise be overloaded. Such a redispatch increases accepted supply in import-constrained nodes and reduces it in export constrained nodes in order to relax intra-zonal congestion. There are alternative ways of compensating producers for their costs associated with these adjustments. The compensation schemes have no direct influence on the cleared zonal prices, but indirectly the details of the design may influence how producers make their offers.

The simplest redispatch is exercised as a command and control scheme: the system operator orders adjustments without referring to the market and all agents are compensated for the estimated cost associated with their adjustments (Krause, 2005). In this paper we instead consider a market oriented redispatch, also called *counter-trading*. This zonal design is used in Britain, in the Nordic countries and it was used in the old Texas design.[5] In these markets a producer's adjustments are compensated in accordance with his stated costs as under discriminatory pricing. Thus the market has a zonal price in the first stage and pay-as-bid pricing in the second stage. We consider two cases: a single shot game where the same bid curve is used in both the first and second stage, and a dynamic game where firms are allowed to submit new bid curves in the second stage. The dynamic model is appropriate if, for example, the first stage represents the day-ahead market and the second stage represents the real-time market.

## 1.2 Comparison of the Three Market Designs

Our analysis considers a general electricity network, which could be meshed, where nodes are connected by capacity constrained transmission lines. We study an idealized market where producers' costs are common knowledge, and demand is certain and inelastic. There is a continuum of infinitesimally small producers that choose their offers in order to maximize their individual payoffs.[6] Subject to the transmission constraints, the system operator accepts offers to minimize total stated production costs, i.e. it clears the market under the assumption that offers reflect true costs. We characterize the Nash equilibrium (NE) of each market design and compare prices, payoffs and efficiencies for the three designs.

In the nodal pricing design, we show that producers maximize their payoffs by simply bidding their marginal costs. Thus, in this case, the accepted offers do in fact maximize short-run social welfare. We refer to these accepted equilibrium offers as the *efficient dispatch* and we call the clearing prices the *network's competitive nodal prices*. We compare this outcome with equilibria in the alternative market designs.

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4. Policy makers' and the industries' critique of the nodal pricing design is summarized, for example, by Alaywan et al. (2004), de Vries et al. (2009), Leuthold et al. (2008), Oggioni and Smeers (2012) and Stoft (1997).

5. Note that Britain is different in that it has pay-as-bid pricing for all accepted bids in the real-time market. The Nordic real-time markets only use discriminatory pricing for redispatches; all other accepted bids are paid a zonal real-time price.

6. The idea to calculate Nash equilibria for a continuum of agents was first introduced by Aumann (1964). The theory was further developed by Green (1984).

For fixed offers, the system operator would increase its profit at producers' expense by switching from nodal to discriminatory pricing. But we show that even if there are infinitely many producers in the market, discriminatory pricing encourages strategic bidding among inframarginal production units. They can increase their offer prices up to the marginal price in their node and still be accepted.[7] In the Nash equilibrium of the pay-as-bid design, accepted production is the same as in the efficient dispatch and all accepted offers are at the network's competitive nodal prices. Thus, market efficiency and payoffs to producers and the system operator are the same as for nodal pricing. As payoffs are identical in all circumstances, this also implies that the long-run effects are the same in terms of investment incentives.

Under our idealized assumptions, the zonal market with counter-trading has the same efficient dispatch as in the two other market designs. We also show that producers buy and sell at the competitive nodal price in the counter-trading stage. Still producers' payoffs are larger under zonal pricing at consumers' and the system operator's expense. The reason is that the two-stage clearing gives producers the opportunity to either sell at the zonal price or at the discriminatory equilibrium price in the second stage, whichever is higher. In addition, even when they are not producing any energy, production units in export-constrained nodes can make money by selling at the uniform zonal price and buying back the same amount at the discriminatory price, which is lower, in the second stage. This increase-decrease game has been observed during the California electricity crisis (Alaywan et al., 2004), it destroyed the initial PJM zonal design, and is present in the UK in the form of large payments to Scottish generators (Neuhoff, Hobbs and Newbery, 2011). Our results show that inc-dec gaming is an arbitrage strategy, which cannot be removed by improving competition in the market. If it is a serious problem, it is necessary to change the market design as in the U.S. We show how producers' profits from the inc-dec game can be calculated for general networks, including meshed networks. Our results for the zonal market are the same for the static game, where the same offer is used in the two stages, and in the dynamic game, where firms are allowed to make new offers in the counter-trading stage.

Additional payments to producers in the zonal market cause long-run inefficiencies; producers overinvest in export-constrained nodes (Dijk and Willems, 2011).[8] Zonal pricing also leads to inefficiencies in the operation of inflexible plants with long ramp-rates, which are not allowed to trade in the real-time market. Related issues are analyzed by Green (2007). In practice nodal pricing is considered superior to the other designs, as it ensures efficient allocation in a competitive market also for uncertain demand and intermittent wind power production; an advantage which is stressed by Green (2010).

The organization of the paper is as follows. In Section 2 we present a simple two–node example illustrating the equilibrium under the nodal pricing. Section 3 discusses our model and in Section 4 we present an analysis of the three congestion management designs. In section 5, market equilibria for the discriminatory and zonal pricing designs are discussed with the means of a simple example. The paper is concluded in section 6, which also briefly discusses how more realistic assumptions would change our results. Three technical lemmas and all proofs are placed in the Appendix.

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7. Related results have been found for theoretical and empirical studies of discriminatory auctions (Holmberg and Newbery, 2010; Evans and Green, 2004). However, previous studies of discriminatory pricing have not taken the network into account.

8. Ruderer and Zöttl (2012) show that zonal pricing in addition leads to inefficient investments in transmission-lines, at least if the zonal market is regulated such that redispatches are compensated according to producers' true costs.

**Figure 1: Equilibrium for Nodal Pricing**
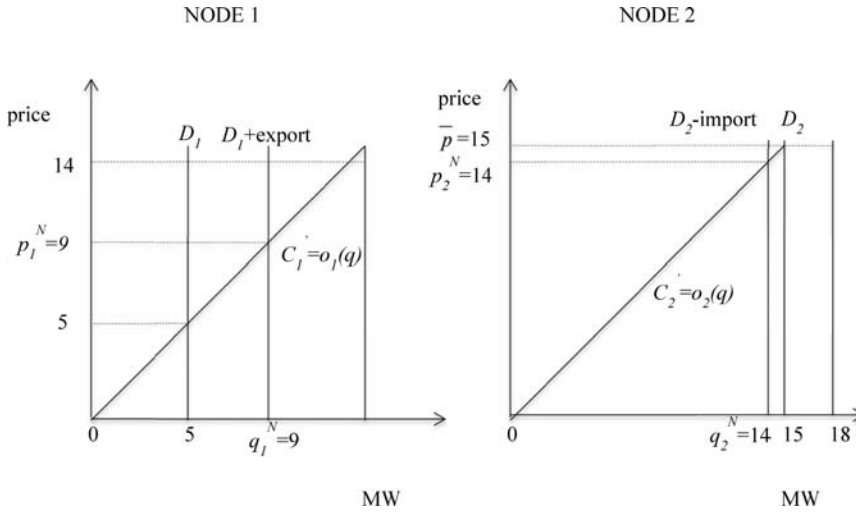


## 2. EXAMPLE—NODAL PRICING

In the following section we describe a simple example of bidding under nodal pricing and the equilibrium outcome of this design. We consider a two-node network with one constrained transmission-line in-between. In both nodes producers are infinitesimally small and demand is perfectly inelastic. For simplicity, we make the following assumptions for each node: the marginal cost is equal to local output and the production capacity is 15 MW. In node 1, demand is 5 MW; in node 2 demand is 18 MW. The transmission line between these nodes is constrained and can carry only 4 MW. Demand in node 2 exceeds its generation possibilities so the missing electricity must be imported from the other node.

With nodal pricing, the equilibrium offers will be as shown in Figure 1. In the first node infinitesimally small producers make nodal offers $o(q)$ at their marginal cost. In order to satisfy local demand and export, 9 MW are going to be dispatched. Out of these, 5 MW will be consumed locally and 4 MW will be exported; the highest possible export level that the transmission line allows for. The marginal cost and nodal price is equal to 9, which corresponds to the total production of this node. In the second node, the nodal price is 14 as there are 14 MW that have to be produced in the second node in order to satisfy demand and the transmission constraint. Production above those marginal costs (9 in node 1 and 14 in node 2) will not be dispatched. All accepted production will be paid the nodal price of the node. The dispatch leads to a socially efficient outcome. We use the superscript $N$ to designate this outcome. We call nodal production and nodal prices of competitive and socially efficient outcomes, the network's efficient dispatch and the network's competitive nodal prices, respectively.

As our analysis will show, the offers in Figure 1 cannot constitute NE in the other two designs. For discriminatory pricing it will be profitable for inframarginal offers to increase their price up to the marginal offer of the node. For zonal pricing, the average demand in the two zones would be 11.5 MW, so 11.5 MW would be accepted in each node at the zonal price 11.5 for the offers in Figure 1. Production would be adjusted in the redispatch stage. However, as it applies discriminatory pricing, it would not influence the payoff of producers that bid their true marginal

cost. Thus producers in the export-constrained node 1 would find it profitable to change their offers downwards. They would like to sell as much as possible at the zonal price and then buy it back at a lower price in the redispatch stage. Producers in the import-constrained node 2 would shift their offers upwards as in the pay-as-bid design, so that all production that is dispatched in the redispatch stage is accepted at the marginal offer of the import constrained node.

## 3. MODEL

The model described in this section is used to evaluate and compare three market oriented congestion management techniques: nodal pricing, pay-as-bid and zonal pricing with counter-trading. We study a general electricity network (possibly meshed) with $n$ nodes that are connected by capacity constrained transmission lines. Demand in a node $i \in \{1, \dots, n\}$ is given by $D_i$, which is certain and inelastic up to a reservation price $\bar{p}$. $C_i'(q_i)$ is the marginal cost of producing $q_i$ units of electricity in node $i$. We assume that the marginal cost is common knowledge, continuous and strictly increasing up to (and beyond) the reservation price.[9] We let $\bar{q}_i > 0$ be the relevant total production capacity in node $i$, which has a marginal cost at the reservation price or lower. Thus we have by construction that $\bar{p} = C_i'(\bar{q}_i)$. Capacity with a marginal cost above the reservation price will not submit any offers.

In each node there is a continuum of infinitesimally small producers. Each producer in the continuum of node $i$ is indexed by the variable $g_i \in [0,1]$. For simplicity, we assume that each producer is only active in one node. Without loss of generality, we also assume that producers are sorted with respect to their marginal cost in each node, such that a producer with a higher $g_i$ value than another producer in the same node also has a higher marginal cost. The relevant total production capacity $\bar{q}_i$ in a node $i$ is divided between the continuum of producers, such that firm $g_i$ in node $i$ has the marginal cost $C_i'(g_i \bar{q}_i)$. Similarly, we let $\hat{o}_i(g_i \bar{q}_i)$ represent the offer price of firm $g_i$ in node $i$.

The system operator's clearing of the real-time market must be such that local net-supply equals local net-exports in each node and such that the physical constraints of the transmission network are not violated. Any set $\{q_i\}_{i=1}^{n}$ of nodal production that satisfies these feasibility constraints is referred to as a feasible dispatch. We say that a dispatch is locally efficient if it minimizes the local production cost in each node for given nodal outputs $\{q_i\}_{i=1}^{n}$, i.e. production units in node $i$ are running if and only if they have a marginal cost at or below $C_i'(q_i)$. We consider a set of demand outcomes $\{D_i\}_{i=1}^{n}$, such that there is at least one feasible dispatch. In principle the network could be a non-linear AC system with resistive losses. But to ensure a unique cost efficient dispatch we restrict the analysis to cases where the feasible set of dispatches is convex. Hence, if two dispatches are possible, then any weighted combination of the two dispatches is also feasible. The feasible set of dispatches is for example convex under the *DC load flow approximation* of general networks with alternating current (Chao and Peck, 1996). [10]

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9. Note that it is possible for a producer to generate beyond the rated power of a production unit. However, it heats up the unit and shortens its lifespan. Thus the marginal cost increases continuously beyond the rated power towards a very high number (above the reservation price) where the unit is certain to be permanently destroyed during the delivery period. Edin (2007) uses a similar marginal cost curve with a similar motivation.

10. Alternating currents (AC) result in a non-linear model of the network. Hence, in economic studies this model is often simplified by a linear approximation called the *direct current (DC) load flow approximation*. In addition to Chao and Peck (1996), it is used, for example, by Schweppe et al. (1988), Hogan (1992), Bjørndal and Jörnsten (2001, 2005, 2007), Glachant and Pignon (2005), Green (2007) and Adler et al. (2008).

The system operator sorts offers in ascending order in case a nodal offer curve $\hat{o}_i(q_i)$ would be locally decreasing. We denote the sorted nodal offer curve by $o_i(q_i)$. The system operator then chooses a feasible dispatch in order to minimize the stated production cost or equivalently to maximize

$$W = -\sum_{i=1}^{n} \underbrace{\int_{0}^{q_i} o_i(y)dy}_{\text{Stated cost}}, \tag{1}$$

which maximizes social welfare if offers would reflect the true costs. Thus, we say that the system operator acts in order to maximize the *stated social welfare* subject to the feasibility constraints.

In a market with nodal pricing the system operator first chooses the optimal dispatch as explained above. All accepted offers in the same node are paid the same nodal price. The nodal price is determined by the node's marginal price, i.e. the highest accepted offer price in the node. We say that marginal prices or nodal prices are locally competitive if the dispatch is locally efficient and the marginal price in each node equals the highest marginal cost for units that are running in the node. An offer at the marginal price of its node is referred to as a marginal offer. In the discriminatory pricing design all accepted offers are paid according to their offer price. This gives producers incentives to change their offers and thereby state their costs differently. Still, the dispatch is determined in the same way; by minimizing stated production cost. In the zonal pricing design with counter-trading, the market is cleared in two stages. First the system operator clears the market disregarding the intra-zonal transmission constraints (constraints inside zones). Next, in case intra-zonal transmission lines are overloaded after the first clearing, there is a redispatch where the system operator increases accepted production in import constrained nodes and reduces it in export constrained nodes. Section 4.3 explains our zonal pricing model in greater detail.

## 4. ANALYSIS

We start our game-theoretical analysis of the three market designs by means of three technical results that we will use in the proofs that follow.

**Lemma 1.** *Assume that offers are shifted upwards (more expensive) in some nodes and shifted downwards (cheaper) in others, then the dispatched production is weakly lower in at least one node with more expensive offers or weakly higher in at least one node with cheaper supply.*

One immediate implication of this lemma is that:

**Corollary 1** (Non-increasing residual demand) *If one producer unilaterally increases/decreases its offer price, then accepted sales in its node cannot increase/decrease.*

The system operator accepts offers in order to minimize stated production costs. Thus for a given acceptance volume in a node, a firm cannot increase its chances of being dispatched by increasing its offer price. Thus Corollary 1 implies that a producer's residual demand is non-increasing. The next lemma outlines necessary properties of a Nash equilibrium.

**Lemma 2.** *Consider a market where an accepted offer is never paid more than the marginal price of its node and never less than its own bid price. In Nash equilibrium, the dispatch must be locally efficient and marginal prices of the nodes are locally competitive.*

### 4.1 Nodal Pricing

Below we prove that the nodal pricing design has at least one NE and that all NE results in the same competitive outcome.[11] It is only offers above and below the marginal prices of nodes that can differ between equilibria.

**Proposition 1** *A market with nodal pricing has one NE where producers offer at their marginal cost. All NE result in the same locally efficient dispatch $\{q_i^N\}_{i=1}^n$ and the same competitive nodal prices $p_i^N = C_i'(q_i^N)$.*

As the system operator clears the market in order to maximize social welfare when offers reveal true costs, we note that the equilibrium dispatch must be efficient. We use the superscript $N$ to designate this socially efficient outcome. We refer to the unique equilibrium outcome as the network's efficient dispatch $\{q_i^N\}_{i=1}^n$ and the network's competitive nodal prices $\{p_i^N\}_{i=1}^n$. Note that as the dispatch is locally efficient, the unique equilibrium outcome exactly specifies which units are running; production units in node $i$ are running if and only if they have a marginal cost at or below $C_i'(q_i^N)$. Schweppe et al. (1988), Chao and Peck (1996) and Hsu (1997) and others outline methods that can be used to calculate efficient dispatches $\{q_i^N\}_{i=1}^n$ for general networks.

Existence of the competitive outcome also indirectly establishes existence of a Walrasian equilibrium, which has previously been proven for radial (Cho, 2003) and meshed networks (Escobar and Jofré, 2008). Proposition 1 proves that all of our NE correspond to the Walrasian equilibrium, so in this sense our NE is equivalent to the Walrasian equilibrium in a market with nodal pricing. The reason is that the infinitesimal producers that we consider are price takers in nodal markets, where all agents in the same node are paid the same market price.

### 4.2 Discriminatory Pricing

Discriminatory pricing is different to nodal pricing in that each agent is then paid its individual offer price rather than a uniform nodal price. Thus, even if agents are infinitesimally small, inframarginal producers can still influence how much they are paid, so they are no longer price takers. This means that the Walrasian equilibrium is not a useful equilibrium concept when studying discriminatory pricing. This is the reason why we instead consider a large game with a continuum of small producers in this paper.

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11. Existence of pure-strategy NE in networks with a finite number of producers is less straightforward. The reason is that a producer in an importing node can find it profitable to deviate from a locally optimal profit maximum by withholding production in order to congest imports and push up the nodal price (Borenstein et al., 2000; Willems, 2002; Downward et al., 2010; Holmberg and Philpott, 2012). Such unilateral deviations are not feasible in a network with infinitesimally small producers, which makes existence of pure-strategy NE more straightforward. Escobar and Jofré (2008) show that networks with a finite number of producers and non-existing pure-strategy NE normally have mixed-strategy NE. Existence of NE in large games with continuous payoffs has been analyzed by Carmona et al. (2009).

**Proposition 2.** *There exist Nash equilibria in a network with discriminatory pricing. All such NE have the following properties:*

1) *The dispatched production is identical to the network's efficient dispatch in each node.*
2) *All production in node i with a marginal cost at or below $C_i'(q_i^N)$ is offered at the network's competitive nodal price $p_i^N = C_i'(q_i^N)$.*
3) *Other offers are not accepted and are not uniquely determined in equilibrium. However, it can, for example, be assumed that they offer at their marginal cost.*

Thus, the discriminatory auction is identical to nodal pricing in terms of payoffs, efficiency, social welfare and the dispatch. As payoffs are identical for all circumstances, this also implies that the long-run effects are the same in terms of investment incentives etc. Note that it is not necessary that rejected offers bid at marginal cost to ensure an equilibrium. As producers are infinitesimally small, it is enough to have a small finite amount of rejected bids at or just above the marginal offer in each node to avoid deviations.

Finally we analyze how contracts influence the equilibrium outcome. We consider forward contracts with physical delivery in a specific node at a predetermined price. For simplicity, we consider cases where each infinitesimally small producer either has no forward sales at all or sells all of its capacity in the forward market for physical delivery in its own node to consumers. In the real-time market, consumers announce how much more power they want to buy in each node, in addition to what they have already bought with contracts, and producers make offers for changes relative to their contractual obligations. The system operator accepts changes in production in order to achieve a feasible dispatch at the lowest possible net-increase in the stated production costs.

**Proposition 3.** *In a real-time market with nodal or discriminatory pricing, the equilibrium dispatch is identical to the network's efficient dispatch and marginal prices of the nodes are competitive, for any set of forward contracts that producers have sold with physical delivery in their own node.*

We will use this result in our analysis of the zonal pricing design, where the first-stage clearing of the zonal market can be regarded as physical forward sales.

### 4.3 Zonal Pricing with Counter-trading

*4.3.1 Notation and assumptions*

Zonal pricing with counter-trading is more complicated than the other two designs and we need to introduce some additional notation before we start to analyze it. The network is divided into zones, such that each node belongs to some zone $k$. We let $Z_k$ be a set with all nodes belonging to zone $k$. To simplify our equations, we number the nodes in a special order. We start with all nodes in zone 1, and then proceed with all nodes in zone 2 etc. Thus, for each zone $k$, nodes are given numbers in some range $\underline{n}_k$ to $\bar{n}_k$. Moreover, inside each zone, nodes are sorted with respect to the network's competitive nodal prices $p_i^N$, which can be calculated for the nodal pricing design, as discussed in Section 4.1. Thus, the cheapest node in zone $k$ is assigned the number $\underline{n}_k$ and the most expensive node in zone $k$ is assigned the number $\bar{n}_k$.

Counter-trading in the second-stage only changes intra-zonal flows. Thus it is important for a benevolent system operator to ensure that the inter-zonal flows are as efficient as possible already after the first clearing. In the Nordic multi-zonal market, system operators achieve this by

announcing a narrow range of inter-zonal flows before the day-ahead market opens. In particular, flows in the "wrong direction", from zones with high prices to zones with low prices, due to loop flows, are predetermined by the system operator. We simplify the zonal clearing further by letting the well-informed system operator set all inter-zonal flows before offers are submitted. Total net-imports to zone $k$ are denoted by $I_k^N$. We make the following assumption for these flows, as our analysis shows that it leads to an efficient outcome:

**Assumption 1:** *The system operator sets inter-zonal flows equal to the inter-zonal flows that would occur for the network's efficient dispatch $\{q_i^N\}_{i=1}^n$. These inter-zonal flows are announced by the system operator before offers are submitted.*

Assumption 1 sets all inter-zonal flows. Thus offers to each zonal market can be cleared separately at a price where zonal net-supply equals zonal net-exports. We assume that the highest potential clearing price is chosen in case there are multiple prices where zonal net-supply equals zonal net-exports.[12] The clearing price $\Pi_k$ in zone $k$ is paid to all production in the zone that is accepted in the zonal clearing. In case intra-zonal transmission-lines are overloaded after the first clearing, there is a redispatch where the system operator increases accepted production in import-constrained nodes and reduces it in export-constrained nodes. We consider a market oriented re-dispatch (counter-trading), so all deviations from the first-clearing are settled on a pay-as-bid basis. In the counter-trading stage, the system operator makes changes relative to the zonal clearing in order to achieve a feasible dispatch at the lowest possible net-increase in stated production costs.

We consider two versions of the zonal design: a one shot game where the same offers are used in the two clearing stages of the market and a dynamic game where agents are allowed to make new offers in the counter-trading stage. The first model corresponds to the old pool in England and Wales, while the latter model could for example be representative of the reformed British market, where producers can first sell power at a uniform zonal price in the day-ahead market and then submit a new bid to the real-time market with discriminatory pricing.[13]

*4.3.2 Analysis*

The equilibrium in a zonal market with counter-trading has some similarities with the discriminatory auction. But the zonal case is more complicated, as the two clearing stages imply that in equilibrium some producers can arbitrage between their zonal and individual (discriminatory) counter-trading prices. Thus producers in nodes with low marginal prices will play the inc-dec game, i.e. sell all their capacity at the higher zonal price and then buy back the capacity at a lower

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12. Normally this choice does not matter for our equilibria. However, it ensures existence of equilibria for degenerate cases when exogenous zonal demand and exogenous net-exports happen to coincide with production capacities in one or several nodes for some zone.

13. The dynamic model could also represent congestion management in the Nordic market, where the system operator does not accept offers in the zonal clearing of the real-time market if these offers will cause intra-zonal congestion that needs to be countertraded in the second-stage. This is to avoid unnecessary costs for the system operator and unnecessary payments to producers. In our model where there is no uncertainty, the zonal day-ahead market then takes the role of the first-stage of the real-time market. The zonal real-time market becomes obsolete as without uncertainty, the day-ahead market has already cleared the zones. In this case offers to the real-time market, which are allowed to differ from day-ahead offers, are only used in the discriminatory counter-trading stage. Proposition 5 shows that under our idealized assumptions switching to the Nordic version of zonal congestion management is in vain; producers still get the same payoffs and the system operator's counter-trading costs are unchanged.

price in the counter-trading stage or produce if the marginal cost is even lower. We consider physical markets. This prevents producers from buying power or selling more than their production capacity in the zonal market. Thus a producer in a node with a marginal price above its zonal price cannot make an arbitrage profit. To maximize their profit in the redispatch stage, bids of dispatched production in such import-constrained nodes are shifted upwards to the node's competitive nodal price, similar to the case with discriminatory pricing.

First we consider a static game where producers cannot make new offers to the counter-trading stage; the same offers are used in the two stages of the zonal market.

**Proposition 4.** *Under Assumption 1 there exists Nash equilibria in a zonal market with counter-trading and the same offers in the zonal and countertrading stages. All of them have the following properties:*

1) *The zonal price in zone k is given by $\Pi_k^* = p_{m(k)}^N$, where:*

$$m(k) \tag{2}$$

$$= \begin{cases} n \in \{\underline{n}_k, \ldots, \bar{n}_k\} : I_k^N + \sum_{i=\underline{n}_k}^{n-1} \bar{q}_i \leq \sum_{i=\underline{n}_k}^{\bar{n}_k} D_i < I_k^N + \sum_{i=\underline{n}_k}^{n} \bar{q}_i & \text{if } \sum_{i=\underline{n}_k}^{\bar{n}_k} D_i < I_k^N + \sum_{i=\underline{n}_k}^{\bar{n}_k} \bar{q}_i \\ \bar{n}_k & \text{if } \sum_{i=\underline{n}_k}^{\bar{n}_k} D_i = I_k^N + \sum_{i=\underline{n}_k}^{\bar{n}_k} \bar{q}_i \end{cases}$$

2) *As in the nodal pricing and pay-as-bid designs, the dispatched production in each node is given by the network's efficient dispatch, $q_i^N$.*
3) *In strictly export-constrained nodes $i \in Z_k$, such that $p_i^N < \Pi_k^*$, production with marginal costs at or above $p_i^N$ are offered at the network's competitive nodal price $p_i^N = C_i'(q_i^N)$. For strictly import-constrained nodes in zone k where $p_i^N > \Pi_k^*$, all production with a marginal cost at or below $C_i'(q_i^N)$ is offered at $p_i^N = C_i'(q_i^N)$.*
4) *Other offers are not uniquely determined in equilibrium. However, it can be assumed that they offer at their marginal cost.*

Equation (2) defines a *marginal node*, where the competitive nodal price equals the zonal price. Next we show that the equilibrium outcome does not change in the dynamic game, where agents are allowed to up-date their offers in the counter-trading stage.

**Proposition 5.** *Under Assumption 1, it does not matter for payoffs or the equilibrium outcome of the zonal market whether producers are allowed to up-date their offers in the counter-trading stage.*

We can now conclude that the dispatch for zonal pricing with counter-trading is the same as for nodal pricing and discriminatory pricing. Thus, in the short run, the designs' efficiencies are equivalent. This also confirms that the system operator should set inter-zonal flows equal to the corresponding flows in the competitive nodal market, as assumed in Assumption 1, if it wants to maximize social welfare. However, it directly follows from Equation (2) and Propositions 4 and 5 that producers in strictly export-constrained nodes receive unnecessarily high payments in a zonal pricing design:

**Corollary 2.** In comparison to nodal pricing, the total extra payoff from the system operator to producers in zone $k$ equals: $\sum_{i=\underline{n}_k}^{m(k)-1}(p_{m(k)}^N - p_i^N)\bar{q}_i$ under Assumption 1.

Even if zonal pricing is as efficient as nodal pricing in the short run, the extra payoffs will cause welfare losses in the long run. Production investments will be too high in strictly export-constrained nodes where $p_i^N < \Pi_k$. In addition, inflexible production that cannot take part in the real-time market are paid the zonal price in the day-ahead market. Thus, the accepted inflexible supply in this market is going to be too high in strictly export-constrained nodes and too low in strictly import-constrained nodes.

## 5. EXAMPLE—DISCRIMINATORY AND ZONAL PRICING

In the following section, we illustrate the equilibria for the discriminatory and zonal pricing designs. The example that we use has an identical structure as the nodal pricing case that we described in section 2. Again, we consider a two-node network with one constrained transmission-line in-between. In both nodes producers are infinitesimally small and demand is perfectly inelastic. In each node the marginal cost is equal to local output and the production capacity is 15 MW. In node 1, demand is 5 MW; in node 2 demand is 18 MW. The transmission line between these nodes is constrained and can carry only 4 MW. Demand in node 2 exceeds its generation possibilities so the missing electricity must be imported from the other node.

The discriminatory design will result in the equilibrium offers presented in Figure 2. In this design, generators are paid according to their bid. Knowing this and having perfect information, producers who want to be dispatched will bid the competitive nodal price of their node, to ensure that they will be dispatched at the highest possible price. Thus, in node 1, they will bid 9 and in node 2 they will bid 14. Producers who do not want to be dispatched may, for example, bid their marginal costs, which are higher than the nodal prices of the respective nodes. The dispatch will be the same as under nodal pricing design. Although producers will have different bidding strategies in both designs, the overall result will be the same. Accepted production will be paid 9 in node 1 and 14 in node 2.

In the zonal design with counter-trading, producers will offer as follows:

Node 1:

Due to transmission constraints, producers in node 1 know that after the two stages, the system operator can accept a maximum of 9 MW in their node. Therefore, producers with a marginal cost at or below the competitive nodal price, offer at or below the competitive nodal price as they will, in any case, be accepted and paid the zonal price, which is 14. The remaining 6 units in node 1 have a marginal cost above the competitive nodal price. They will bid low in order to be accepted in the first stage and be paid the zonal price of 14. But due to the transmission constraint, they will have to buy back their supply at their own bidding price in the second round. As they are interested in maximizing their profit, they want this price difference to be as large as possible, as long as they will not be chosen to produce. Therefore, they bid the competitive nodal price 9 so that they will be "paid" not to produce and get $14-9=5$ (the rectangle area in the Figure 4). There are no profitable deviations from these bids for producers from node 1. In particular, we note that no infinitesimally small producer in node 1 can unilaterally increase the zonal price

**Figure 2: Equilibrium for Discriminatory Pricing**
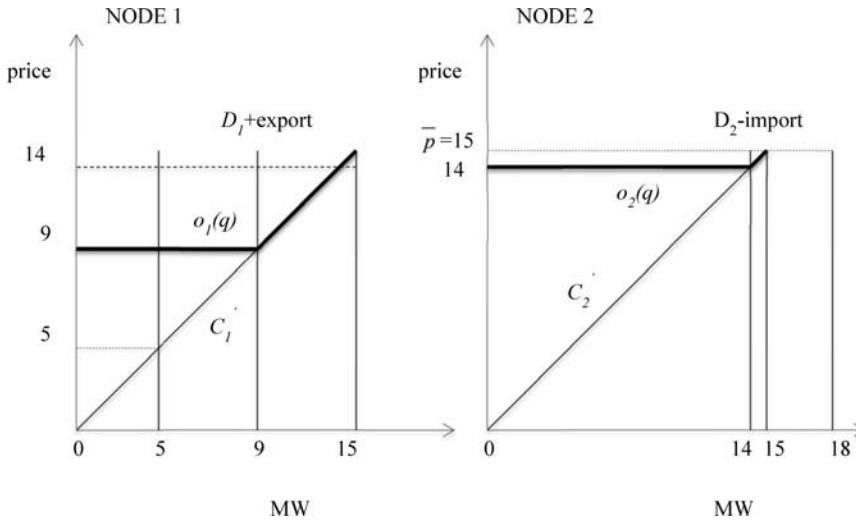


**Figure 3: Zonal Offer in Equilibrium for Zonal Pricing with Counter-trading**
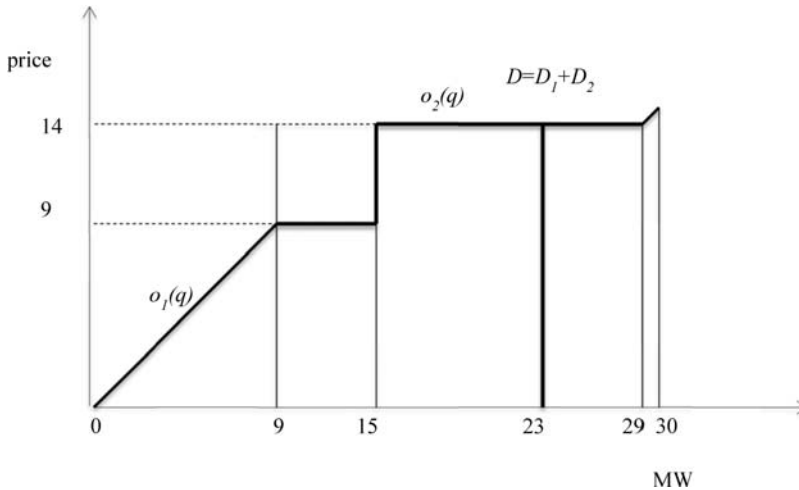


at stage 1 above 14, as there are 6 units (in node 2) that offer their production at the price 14 without being accepted in the zonal market.

Node 2:

Due to the transmission constraint, producers in node 2 know that the system operator needs to dispatch at least 14 units of electricity in their node after the two stages. Thus, all low-cost generators who want to be dispatched know that all offers at or below 14, the competitive nodal price of node 2, will be accepted. 8 units are accepted in the zonal clearing and another 6 units are accepted in the counter-trade stage. The latter units are paid as bid and accordingly, they maximize their profit by offering their supply at 14, the highest possible price for which they are going to be accepted. Producers that do not want

**Figure 4: Nodal Offers in Equilibrium for Zonal Pricing with Counter-trading**
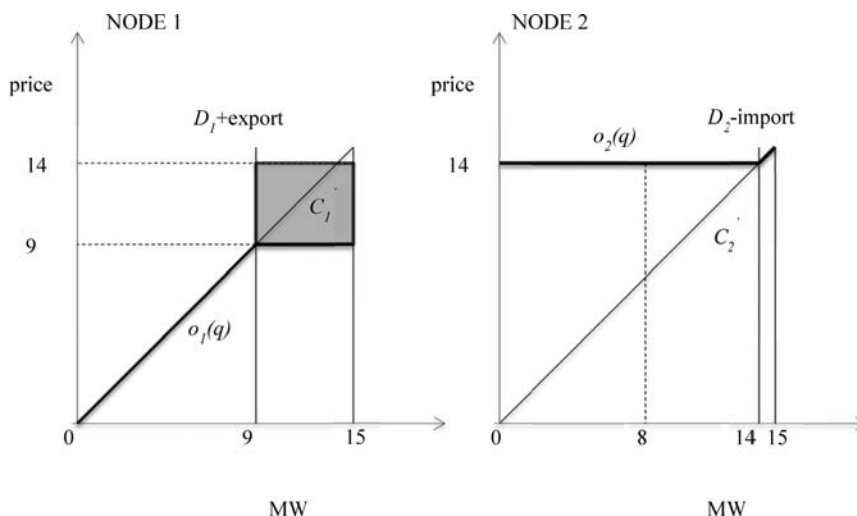


to be dispatched at all will bid above 14, for example their marginal cost. In this way, 14 units will be produced in node 2. There are no profitable deviations from these strategies for producers in node 2.

A comparison of these two examples and the nodal pricing example in Section 2 illustrates that although the bidding strategies are different, the dispatch is the same in all scenarios. However, the last design–zonal pricing with counter-trading–results in additional payments that affect the long-term investment incentives.

It is interesting to note that the zonal price in our example is weakly higher than the nodal prices in both nodes. This is always the outcome in two-node networks where the production capacity in the cheapest node is not sufficient to meet the total demand, so that it is the marginal cost in the most expensive node that sets the zonal price. The system operator will typically use tariffs to pass its counter-trading cost on to the market participants, so it is actually quite plausible that switching to nodal pricing will lower the cost for all electricity consumers, including the ones in the high cost node.

## 6. CONCLUSIONS AND DISCUSSION

We consider a general electricity network (possibly meshed), where nodes are connected by capacity constrained transmission lines. In our game-theoretical model producers are infinitesimally small and demand is certain and inelastic. We find that the three designs, nodal, zonal with countertrading and discriminatory pricing, lead to the same socially efficient dispatch. In addition, payoffs are identical in the pay-as-bid and nodal pricing designs. However, in the design with zonal pricing and countertrading, there are additional payments from the system operator to producers who can make money by playing the infamous inc-dec game. It does not matter for our results whether we consider a static game where producers' bids are the same in the zonal and counter-trading stages or a dynamic game where producers are allowed to update their offer curves in the counter-trading stage.

Similar to Dijk and Willems' (2011) two-node model, our results for the zonal market imply that producers overinvest in export-constrained nodes. While zonal pricing is good for producers, consumers would gain overall from a switch from zonal to nodal pricing. In two-node markets, it is normally the case that all consumers (also the ones in the most expensive node) would gain from a switch to nodal pricing. In addition to the inefficiencies implied by our model, zonal pricing also leads to inefficiencies in the operation of inflexible plants with long ramp-rates. They are not allowed to trade in the real-time market, so they have to sell at the zonal price in the day-ahead market. The consequence is that too much inflexible production is switched on in export-constrained nodes, where the competitive nodal price is below the zonal price, and too little in import constrained nodes, where the competitive nodal price is above the zonal price. Related issues are analyzed by Green (2007).

Another result from our analysis is that there is a significant number of firms that make offers exactly at the marginal prices of the nodes in the zonal and pay-as-bid designs, which is not necessarily the case under nodal pricing. This supports the common view that the zonal design is more liquid. Although, the standard motivation for this is that the zonal design has less market prices and thus fewer products to trade, and hence liquidity can be concentrated on these. Still it is known from PJM that it is also possible to have a liquid market with nodal pricing (Neuhoff and Boyd, 2011).

However increased liquidity can have more drawbacks than advantages. As illustrated by Anderson et al. (2009), the elastic offers, especially in the pay-as-bid design but also in the zonal design, mean that getting its offer slightly wrong can have a huge effect on a firm's dispatch. This increases the chances of getting inefficient dispatches when demand or competitors' output is uncertain, while the efficiency of the nodal pricing design is more robust to these uncertainties. Similarly, Green (2010) stresses the importance of having designs that can accommodate uncertainties from intermittent power.

There are other drawbacks with the zonal design. We consider a benevolent system operator that uses counter-trading to find the socially optimal dispatch. However, even if counter-trading is socially efficient, it is costly for the system operator itself. Thus strategic system operators have incentives to find the feasible dispatch that minimizes counter-trading costs. In practice, counter-trading is therefore likely to be minimalistic and less efficient than in our framework. Moreover, Bjørndal et al. (2003) and Glachant and Pignon (2005) show that network operators have incentives to manipulate inter-zonal flows in order to lower the counter-trading cost (and market efficiency) further. In our analysis we assume that the system-operator has full control of the system and that it can set inter-zonal flow as efficiently as under nodal pricing, but in practice market uncertainty, coordination problems and imperfect regulations lead to significantly less efficient cross-border flows (Leuthold, 2008; Neuhoff, et al., 2011; Ogionni and Smeers 2012). Studies by Hogan (1999), Harvey and Hogan (2000), and Green (2007) indicate that nodal pricing is also better suited to prevent market power.

## ACKNOWLEDGMENTS

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## APPENDIX A: TECHNICAL LEMMAS

**Lemma 3.** *$m(k)$ is uniquely defined by Equation (2).*

**Proof:** We first note that the network's efficient dispatch is feasible as the inter-zonal flows are efficient, i.e. $\sum_{i=\underline{n}_k}^{\bar{n}_k} D_i = I_k^N + \sum_{i=\underline{n}_k}^{\bar{n}_k} q_i^N$. Thus $\sum_{i=\underline{n}_k}^{\bar{n}_k} D_i \leq I_k^N + \sum_{i=\underline{n}_k}^{\bar{n}_k} \bar{q}_i$. We have $m(k) = \bar{n}_k$ if $\sum_{i=\underline{n}_k}^{\bar{n}_k} D_i = I_k^N + \sum_{i=\underline{n}_k}^{\bar{n}_k} \bar{q}_i$. Otherwise we have $I_k^N + \sum_{i=\underline{n}_k}^{n_k-1} \bar{q}_i < \sum_{i=\underline{n}_k}^{\bar{n}_k} D_i < I_k^N + \sum_{i=\underline{n}_k}^{\bar{n}_k} \bar{q}_i$. Moreover, $I_k^N + \sum_{i=\underline{n}_k}^{n-1} \bar{q}_i$ is strictly increasing in $n$, because $\bar{q}_i > 0$. Thus Equation (2) always has a unique solution. $\blacksquare$

The following two technical lemmas are used to prove that all Nash equilibria must result in the same dispatch.

**Lemma 4.** *If there is a set of nodal offer functions $\{\hat{o}_i^*(q)\}_{i=1}^n$ (not necessarily increasing) that results in a locally efficient dispatch with the nodal output $\{q_i^*\}_{i=1}^n$ and locally competitive marginal prices, then any set of strictly increasing nodal offer functions $\{\hat{o}_i(q)\}_{i=1}^n$, such that $\hat{o}_i(q_i^*) = \hat{o}_i^*(q_i^*) \; \forall i \in \{1,\dots,n\}$, will result in the same dispatch.*

**Proof:** First, consider the case when offers $\{\hat{o}_i^*(q)\}_{i=1}^n$ are also strictly increasing in output. In this case, the objective function (stated welfare) is strictly concave in the supply, $q_i$. Moreover, the set of feasible dispatches is by assumption convex in our model. Thus, it follows that the objective function has a unique local extremum, which is a global maximum (Gravelle and Rees, 1992). Thus the system operator's dispatch can be uniquely determined. It follows from the necessary Lagrange condition that the unique optimum is not influenced by changes in node $i$'s offers below and above the quantity $q_i^*$, as long as offers are strictly increasing in output. Thus the dispatch must be the same for any set of strictly increasing nodal offer functions $\{\hat{o}_i(q)\}_{i=1}^n$, such that $\hat{o}_i(q_i^*) = \hat{o}_i^*(q_i^*) \; \forall i \in \{1,\dots,n\}$.

With perfectly elastic segments in the offer curves $\{o_i^*(q)\}_{i=1}^n$ there are output levels, for which $o_i^{*\,\prime}(q) = 0$ in some node $i$. This means that the objective function is no longer strictly concave in the supply. However, one can always construct strictly increasing curves that are arbitrarily close to curves with perfectly elastic segments. Moreover, the system operator's objective function is continuous in offers. Thus, we can use the same argument as above with the difference that the system operator may sometimes have multiple optimal dispatches, in addition to the dispatch above, for a given set of offer curves $\{\hat{o}_i^*(q)\}_{i=1}^n$.[14] However, the same dispatch as above is pinned down by the additional conditions that the dispatch is locally efficient and marginal prices locally competitive.

Finally, we realize that there could be cases with non-monotonic offers $\{\hat{o}_i^*(q)\}_{i=1}^n$. However, the dispatch is locally efficient and marginal prices locally competitive, so such offers would have to satisfy the following properties $\hat{o}_i^*(q) \le \hat{o}_i^*(q_i^*)$ for $q \le q_i^*$ and $\hat{o}_i^*(q) \ge \hat{o}_i^*(q_i^*)$ for $q \ge q_i^* \; \forall i \in \{1,\dots,n\}$. Thus as the system operator sorts offers into ascending order, we can go through the arguments above for sorted offers and conclude that the statement must hold for such cases as well. ∎

**Lemma 5.** *If two sets of nodal offer functions both result in a locally efficient dispatch with locally competitive marginal prices, then the two resulting dispatches must be identical.*

**Proof:** Make the contradictory assumption that there are two pairs of offer functions with a corresponding dispatch, $\{\{\hat{o}_i^*(q)\}_{i=1}^n, \{q_i^*\}_{i=1}^n\}$ and $\{\{\hat{o}_i^\times(q)\}_{i=1}^n, \{q_i^\times\}_{i=1}^n\}$, that satisfy the stated properties, except that $\{q_i^*\}_{i=1}^n \ne \{q_i^\times\}_{i=1}^n$. Lemma 4 states how these offers can be adjusted into strictly increasing offer curves without changing the dispatch. We make such adjustments to get two sets of adjusted nodal offer functions, $\{\hat{o}_i^*(q)\}_{i=1}^n$ and $\{\hat{o}_i^\times(q)\}_{i=1}^n$ that are identical in nodes with the same dispatch and non-crossing in the other nodes.

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14. Multiple optimal dispatches for example occur if several units in a node have the same stated marginal cost and some but not all of these units are accepted in a dispatch that minimizes stated production costs.

By assumption we have $\hat{o}_i^*(q_i^*) = C_i'(q_i^*)$ and $\hat{o}_i^\times(q_i^\times) = C_i'(q_i^\times)$. The node's marginal cost curve is strictly increasing in output. Thus adjusted offers $\{\hat{o}_i^*(q)\}_{i=1}^n$ must be above (more expensive) compared to adjusted offers $\{\hat{o}_i^\times(q)\}_{i=1}^n$ in all nodes where $q_i^* > q_i^\times$. Similarly, adjusted offers $\{\hat{o}_i^*(q)\}_{i=1}^n$ must be below (cheaper) compared to adjusted offers $\{\hat{o}_i^\times(q)\}_{i=1}^n$ in all nodes where $q_i^* < q_i^\times$. However, this would violate Lemma 1. Thus, the dispatches $\{q_i^*\}_{i=1}^n$ and $\{q_i^\times\}_{i=1}^n$ must be identical. ∎

## APPENDIX B: OTHER PROOFS

### Proof of Lemma 1

We let the *old dispatch* refer to the feasible dispatch $\{q_i^{old}\}_{i=1}^n$ that maximized stated social welfare at old offers when supply in node $i$ is given by $o_i(q_i)$. Let $\Delta o_i(q_i)$ denote the shift of the supply curve, so that $o_i(q_i) + \Delta o_i(q_i)$ is the new supply curve in node $i$. The *new dispatch* refers to the feasible dispatch $\{q_i^{new}\}_{i=1}^n$ that maximizes stated social welfare for new offers. Thus for new offers, $o_i(q_i) + \Delta o_i(q_i)$, the new dispatch $\{q_i^{new}\}_{i=1}^n$ should result in a weakly higher social welfare than the old dispatch $\{q_i^{old}\}_{i=1}^n$, i.e.

$$-\sum_{i=1}^n \int_0^{q_i^{new}} (o_i(x) + \Delta o_i(x))dx \geq -\sum_{i=1}^n \int_0^{q_i^{old}} (o_i(x) + \Delta o_i(x))dx. \tag{3}$$

Now, make the contradictory assumption that in comparison to the old dispatch, the new dispatch has strictly more production in all nodes where offers have been shifted upwards (more expensive) and strictly less production in all nodes where offers have been shifted downwards (cheaper). Thus $q_i^{new} > q_i^{old}$ when $\Delta o_i(q_i) \geq 0$ with strict inequality for some $q_i \in (0, q_i^{new})$, and $q_i^{new} < q_i^{old}$ when $\Delta o_i(q_i) \leq 0$ with strict inequality for some $q_i \in (0, q_i^{old})$, so that

$$\sum_{i=1}^n \int_0^{q_i^{new}} \Delta o_i(x)dx > \sum_{i=1}^n \int_0^{q_i^{old}} \Delta o_i(x)dx. \tag{4}$$

But summing Equation (3) and Equation (4) yields

$$-\sum_{i=1}^n \int_0^{q_i^{new}} o_i(x)dx > -\sum_{i=1}^n \int_0^{q_i^{old}} o_i(x)dx, \tag{5}$$

which is a contradiction since, by definition, the old dispatch $\{q_i^{old}\}_{i=1}^n$ is supposed to maximize stated welfare at old offers. ∎

### Proof of Lemma 2

The statement follows from that: 1) offers cannot be dispatched at a price below their marginal cost in equilibrium, and that 2) all offers from production units with a marginal cost at or below the marginal price of a node must be accepted in equilibrium. If 1) did not hold for some firm then it would be a profitable deviation for the firm to increase its offer price to its marginal cost. 2) follows from that there would otherwise exist some infinitesimally small producer in the node with a marginal cost below the marginal price, but whose offer is not dispatched. Thus, it would be a profitable deviation for such a producer to slightly undercut the marginal price and we know from

Corollary 1 that such a deviation will not decrease the dispatched production in its node, so the revised offer will be accepted. ■

**Proof of Proposition 1**

We note that the objective function (stated welfare) in Equation (1) is continuous in the nodal output $q_i$ when offers are at the marginal cost. Moreover, the feasible set (the set of possible dispatches) is closed, bounded (because of capacity constraints) and non-empty. Thus, it follows from Weierstrass' theorem that there always exists an optimal feasible dispatch when offers reflect true costs (Gravelle and Rees, 1992).

Next, we note that no producer has a profitable deviation from the competitive outcome. Marginal costs are continuous and strictly increasing. Hence, it follows from Corollary 1 that no producer with an accepted offer can increase its offer price above the marginal price of the node and still be accepted, as its offer price would then be above one of the previously rejected offers in the same node.[15] No producer with a rejected offer would gain by undercutting the marginal price, as the changed offer would then be accepted at a price below its marginal cost. Thus, there must exist an NE where all firms offer to produce at their marginal cost. Offers above and below the marginal price of a node can differ between equilibria. But it follows from Lemma 2 and Lemma 5 in Appendix A that all NE must have the same locally efficient dispatch and the same locally competitive marginal prices, so nodal prices, which are set by marginal prices, must also be the same. ■

**Proof of Proposition 2**

Proposition 1 ensures existence of the network's efficient dispatch and competitive nodal prices. Both nodal and discriminatory pricing are markets where an accepted offer is never paid more than its node's marginal price and never less than its own bid price, so in both cases the equilibrium dispatch must be locally efficient and marginal prices of the nodes are competitive in equilibrium, because of Lemma 2. Thus statement 1) follows from Lemma 5 in Appendix A. In a discriminatory market it is profitable for a producer to increase the price of an accepted offer until it reaches the marginal price of its node, which gives statement 2). Finally, we realise that there are no profitable deviations from the stated equilibrium if rejected offers are at their marginal cost. ■

**Proof of Proposition 3**

We note that the stated production cost of contracted sales is a constant. Thus we can add it to the objective function of the system operator's optimization problem without influencing the optimal dispatch. The set of feasible dispatches is not influenced by producers' forward sales. Thus to solve for the optimal dispatch we can add producers' forward sales to their offered quantities, so that offers include contracted quantities instead of being net of contracts, and then solve for the feasible dispatch that minimizes the total stated production costs as defined by Equation (1). Rewriting the dispatch problem in this way, implies that Lemma 1, Corollary 1, Lemma 4 and Lemma 5 in Appendix A also apply to situations with contracts. Thus the stated result would follow if we can prove that the dispatch must be locally efficient and marginal prices of the nodes are competitive in equilibrium, also for contracts. Similar to the proof of Lemma 2, this follows from that: 1) a

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15. Also note that the last unit in a node cannot increase its offer above its marginal cost due to the reservation price $\bar{p} = C'(\bar{q}_i)$.

production unit cannot be dispatched at a real-time price below its marginal cost in equilibrium, and that 2) all production units with a marginal cost at or below the marginal real-time price of its node must be dispatched in equilibrium. The proof of Lemma 2 explains why 1) must hold for uncontracted firms. If 1) would not hold for a contracted firm, then it would be a profitable deviation for the firm to increase its offer price (to buy back the contract and avoid being dispatched) to a price above the marginal real-time price and below its marginal cost. It follows from Corollary 1 that such a unilateral deviation cannot increase the nodal production in the contracted firm's node. Thus its offer to buy back the contract is accepted at a price below its marginal cost, which is cheaper than to follow the contracted obligation and produce at marginal cost. 2) follows from that there would otherwise exist some infinitesimally small producer in the node with a marginal cost below the marginal price, but whose offer is not dispatched. We already know from the proof of Lemma 2 that such a producer would find a profitable deviation if it was uncontracted. We also realize that a producer that has sold its production forward and that has a marginal cost below the marginal price would lose from bidding above the marginal price (to buy back the contract), so that its unit is not dispatched. It would be a profitable deviation for such a producer to lower its bid to its marginal cost. It follows from Corollary 1 that such a change would not decrease accepted production. Thus it increases its payoff by at least the difference between its nodal marginal price and its marginal cost. ■

### Proof of Proposition 4

Existence of a competitive equilibrium in the nodal design follows from Proposition 1. Assumption 1 restricts inter-zonal flows to be efficient. However, we realize from the proof of Proposition 3 that this extra constraint does not change the statement in Proposition 3. A producer's accepted offer in the zonal market is equivalent to a forward position with physical delivery in its node. Thus it follows from Proposition 3 that, independent of the zonal clearing, the equilibrium dispatch is identical to the network's efficient dispatch and marginal prices of the nodes are competitive in the counter-trading stage. This gives the unique dispatch $\{q_i^N\}_{i=1}^n$ as stated in 2). The counter-trading stage uses discriminatory pricing, but all agents want to trade at the best price possible, so all accepted offers in the counter-trading stage are marginal offers at the network's competitive nodal prices.

Consider a zone $k$ with its associated nodes $n \in Z_k$ or equivalently $n \in \{\underline{n}_k, \ldots, \bar{n}_k\}$. A node inside zone $k$ where the network's competitive nodal price $p_i^N$ is strictly below the zonal price $\Pi_k$ is referred to as a strictly export constrained node. Price-taking producers in such nodes want to sell as much production as they can at the zonal price, and then buy back production in the discriminatory counter-trading stage at the lower price $p_i^N$ or produce at an even lower marginal cost. Thus all capacity in a strictly export constrained node $i$ is offered at or below $p_i^N < \Pi_k$. As the real-time market is physical, producers in strictly import-constrained nodes of zone $k$ (where the network's competitive nodal price $p_i^N$ is strictly above the zonal price $\Pi_k$) are not allowed to first buy power at a low price in the zonal market and then sell power at $p_i^N$ in the counter-trading stage. Thus they neither buy nor sell any power in the zonal market, so they make offers above $\Pi_k$. We can conclude from the above reasoning that a marginal offer at the zonal price cannot come from a production unit that is located in a node that is strictly export or import constrained. In equilibrium there must be at least one marginal node $m$ with $p_m^N = \Pi_k$. Recall that nodes have been sorted with respect to competitive nodal prices and that that the highest clearing price is chosen in case there are multiple prices where zonal net-supply equals zonal net-exports. Thus we can define one mar-

ginal node by Equation (2).[16] It follows from Lemma 3 that this equation uniquely sets the zonal price $\Pi_k = p^N_{m(k)}$.

Offers in strictly import constrained nodes, which are above the zonal price, are never accepted in the first stage of the zonal market. For these nodes, it is the rules of the counter-trading stage that determine optimal offer strategies. Thus, the auction works as a discriminatory auction, and we can use the same arguments as in Proposition 2 and Proposition 3 to prove the second part of statement 3).

Production units in a strictly export-constrained node that have a higher marginal cost than their competitive nodal price can sell their power in the zonal market at the zonal price and then buy it back at a lower offer price in the counter-trade stage. Thus, to maximize profits this power is offered at the lowest possible price, for which offers are not dispatched, i.e. at the marginal price of the node. This gives the first part of statement 3). Non-dispatched production units would not gain by undercutting the marginal price. Offers that are dispatched in strictly export-constrained nodes are paid the zonal price. It is not possible for one of these units to increase its offer price above $p^N_i < \Pi_k$ and still be dispatched, as non-dispatched units in such nodes offer at $p^N_i$. Moreover, it is weakly cheaper for dispatched units to produce instead of buying back power at $p^N_i$. Thus, they do not have any profitable deviations. Accordingly, the stated offers must constitute a Nash equilibrium. ■

## Proof of Proposition 5

We solve the two-stage game by backward induction. Thus we start by analysing the countertrading stage. A producer's accepted offer in the zonal market is equivalent to a forward position with physical delivery in its node. Thus it follows from Proposition 3 that, independent of the zonal clearing, the equilibrium dispatch is identical to the network's efficient dispatch and marginal prices of the nodes are competitive in the counter-trading stage. The counter-trading stage uses discriminatory pricing, but all agents want to trade at the best price possible, so all accepted offers in the counter-trading stage are marginal offers at the network's competitive nodal prices.

We calculate a subgame perfect Nash equilibrium of the game, so rational agents realise what the outcome of the second-stage is going to be, and make offers to the zonal market in order to maximize profits. Thus, similar to the one-stage game, all production capacity in strictly export-constrained nodes $i \in Z_k$, such that $p^N_i < \Pi_k$, is sold at the zonal price. As before, production capacity in strictly import constrained nodes maximize their payoff by selling no power in the zonal market; all production that is dispatched in strictly import constrained nodes is accepted in the counter-trading stage. As in the one-stage game, the zonal price in zone $k$ must be set by the marginal price of some marginal node $m$ as defined in Equation (2). Otherwise there must be some offer to the zonal market from a production unit in a strictly export constrained node (with $p^N_i < \Pi_k$) that is rejected, and which would find it profitable to slightly undercut the zonal price. All production units that are dispatched in marginal nodes are sold at the zonal price. As in the one-stage game, there are always rejected offers from units in marginal nodes that can be placed at or just above the zonal price. This rules out that profitable deviations for production units in marginal nodes. Thus all agents get the same payoffs as the game in Proposition 4, where the same offers were used in the zonal and countertrading stages. ■

16. It is possible that nodes with numbers adjacent to $m(k)$ have the same competitive nodal prices as node $m(k)$, but it will not change the analysis. It is enough to find one marginal node to determine the zonal price. As an example, it follows from Proposition 1 and our cost assumptions that in the special case when zonal demand equals the zonal production capacity plus efficient imports, then the competitive nodal price equals the price cap in all nodes. Thus any node could be chosen to be the marginal node, but $\bar{n}_k$ is the most natural extension of the first part of Equation (2).