A Real Options Analysis of the Effects of Oil Price Uncertainty and Carbon Taxes on the Optimal Timing of Oil Field Decommissioning

Online Supplementary Material

Appendix I: Empirical estimates for model calibration

Empirical estimates of the baseline values of the three sources of oil price uncertainty in equation (4) of the paper are required for the overall model calibration. These are (1) the degree of oil price volatility, σ ; (2) the long-run equilibrium oil price, p_{eqm} ; and (3) the speed of reversion of oil prices to their long-run equilibrium price, α . To estimate these parameters, we follow Smith (2010) to reformulate the mean reversion process in equation (4) of the paper and express it in the generic form shown in equation (i), as captured in equation (ii);

$$\boldsymbol{p}_{t+1} = \beta_0 + \beta_1 \cdot \boldsymbol{p}_t + \boldsymbol{\epsilon}_t \tag{i}$$

$$\boldsymbol{p}_{t+1} = (1 - e^{-\alpha \Delta t}) \cdot \boldsymbol{p}_{eqm} + e^{-\alpha \Delta t} \cdot \boldsymbol{p}_t + \sigma \sqrt{\frac{(1 - e^{-2\alpha \Delta t})}{2\alpha}} \cdot \varepsilon_t$$
(ii)

In equations (ii), p_{t+1} and p_t are dependent and independent variables respectively, analogous to the expression in equation (i). Given an oil price series p_t , equation (i) can be estimated using an Ordinary Least Squares (OLS) estimator. By superimposing equations (i) and (ii), the empirical estimates of the three parameters can be calculated from the OLS estimation results as follows (Smith, 2010);

$$\hat{\alpha} = -\frac{\ln(\beta_1)}{\Delta t} \tag{iii}$$

$$\hat{p}_{eqm} = \frac{\beta_0}{(1 - \beta_1)} \tag{iv}$$

$$\hat{\sigma} = sd(\epsilon_t) \sqrt{\frac{2\hat{\alpha}}{(1 - e^{-2\hat{\alpha}\Delta t})}}$$
 (v)

We treat the above empirical estimates of the long-run equilibrium price \hat{p}_{eqm} and the oil price volatility $\hat{\sigma}$ as the baseline values for model calibration. We use US EIA (2021) annual average Europe Brent spot price (FOB \$/bbl) series for the period 1987 – 2020 to carry out the above estimation (see Appendix II below for a plot of the data). US EIA (2021) calculate the annual price series from daily data by taking an unweighted average of the daily closing spot prices of oil over each year. Estimation results for equation (i) are reported in Appendix III below. We however do not use our empirical estimate of the price reversion speed $\hat{\alpha}$ as our baseline value. As Smith (2010) and Kruse and Wegener (2020) note, the OLS estimator for this quantity has a downward bias. We therefore select the baseline value of the speed of reversion from a range of reported values from the empirical literature that use more sophisticated techniques to mitigate this bias.

Appendix II – Oil price data

Data is the annual average US EIA (2021) Europe Brent spot price (FOB \$/bbl) series for the period 1987 – 2020.



Appendix III – Estimating parameters of the O-U price reversion

process

 p_t is the price of oil (See equation (i) above). Data is the annual average US EIA (2021) Europe Brent spot price (FOB \$/bbl) series for the period 1987 – 2020, as shown in Appendix II above.

Dependent variable: p_{t+1}	
Independent variables	Coeffcient
p_t	0.879***
	(0.081)
Constant	6.306
	(4.554)
R-squared	0.636
Number of observations	33
Standard amount in namethanan	

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Appendix IV – Empirical results from simulated data

Independent variablesCoeffcientEquilibrium oil price 0.236^{***} Equilibrium oil price (0.000) Oil price volatility -0.051^{***} Mean reversion speed 3.281^{***} Equilibrium oil price × Oil price volatility -0.001^{***} Equilibrium oil price × Oil price volatility -0.055^{***} Oil price volatility × Mean reversion speed -0.055^{***} Oil price volatility × Mean reversion speed -0.044^{***} Equilibrium oil price × Oil price volatility × Mean reversion speed -0.044^{***} Oil price volatility × Mean reversion speed 0.003^{***} Constant 9.900^{***}	Dependent variable: Optimal decommissioning period		
Equilibrium oil price 0.236^{***} (0.000)Oil price volatility -0.051^{***} (0.001)Mean reversion speed 3.281^{***} (0.016)Equilibrium oil price × Oil price volatility -0.001^{***} (0.000)Equilibrium oil price × Mean reversion speed -0.055^{***} (0.000)Oil price volatility × Mean reversion speed -0.044^{***} (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed 0.003^{***} (0.000)Constant 9.900^{***}	Independent variables	Coeffcient	
Oil price volatility (0.000) $-0.051***$ (0.001) Mean reversion speed 3.281^{***} (0.016) Equilibrium oil price × Oil price volatility -0.001^{***} (0.000) Equilibrium oil price × Mean reversion speed -0.055^{***} (0.000) Oil price volatility × Mean reversion speed -0.044^{***} (0.001) Equilibrium oil price × Oil price volatility × Mean reversion speed -0.044^{***} (0.001) Equilibrium oil price × Oil price volatility × Mean reversion speed 0.003^{***} (0.000) Constant 9.900^{***}	Equilibrium oil price	0.236***	
Oil price volatility -0.051^{***} (0.001)Mean reversion speed 3.281^{***} (0.016)Equilibrium oil price × Oil price volatility -0.001^{***} (0.000)Equilibrium oil price × Mean reversion speed -0.055^{***} (0.000)Oil price volatility × Mean reversion speed -0.044^{***} (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed -0.044^{***} (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed 0.003^{***} (0.000)Constant 9.900^{***}		(0.000)	
Mean reversion speed (0.001) Equilibrium oil price × Oil price volatility -0.001^{***} Equilibrium oil price × Mean reversion speed -0.055^{***} Oil price volatility × Mean reversion speed -0.044^{***} Equilibrium oil price × Oil price volatility × Mean reversion speed -0.044^{***} Constant 9.900^{***}	Oil price volatility	-0.051***	
Mean reversion speed3.281***Equilibrium oil price × Oil price volatility-0.001***Equilibrium oil price × Mean reversion speed-0.055***Oil price volatility × Mean reversion speed-0.044***Equilibrium oil price × Oil price volatility × Mean reversion speed0.003***Constant9.900***		(0.001)	
Equilibrium oil price × Oil price volatility(0.016)Equilibrium oil price × Mean reversion speed-0.001***Oil price volatility × Mean reversion speed-0.055***Oil price volatility × Mean reversion speed-0.044***Equilibrium oil price × Oil price volatility × Mean reversion speed0.003***Constant9.900***	Mean reversion speed	3.281***	
Equilibrium oil price × Oil price volatility-0.001*** (0.000)Equilibrium oil price × Mean reversion speed-0.055*** (0.000)Oil price volatility × Mean reversion speed-0.044*** (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed0.003*** (0.000)Constant9.900***	-	(0.016)	
Equilibrium oil price × Mean reversion speed(0.000) -0.055*** (0.000)Oil price volatility × Mean reversion speed-0.044*** (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed0.003*** (0.000)Constant9.900***	Equilibrium oil price \times Oil price volatility	-0.001***	
Equilibrium oil price × Mean reversion speed-0.055*** (0.000)Oil price volatility × Mean reversion speed-0.044*** (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed0.003*** (0.000)Constant9.900***		(0.000)	
Oil price volatility × Mean reversion speed(0.000) -0.044*** (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed0.003*** (0.000) 9.900***	Equilibrium oil price \times Mean reversion speed	-0.055***	
Oil price volatility × Mean reversion speed-0.044*** (0.001)Equilibrium oil price × Oil price volatility × Mean reversion speed0.003*** (0.000)Constant9.900***		(0.000)	
Equilibrium oil price × Oil price volatility × Mean reversion speed(0.001) 0.003*** (0.000) 9.900***	Oil price volatility \times Mean reversion speed	-0.044***	
Equilibrium oil price × Oil price volatility × Mean reversion speed0.003***(0.000)9.900***		(0.001)	
Constant (0.000) 9.900***	Equilibrium oil price \times Oil price volatility \times Mean reversion speed	0.003***	
Constant 9.900***		(0.000)	
	Constant	9.900***	
(0.010)		(0.010)	
R-squared 0.636	R-squared	0.636	
Number of observations, millions 18	Number of observations, millions	18	

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1