

# A GAME THEORETICAL SEASONALIZATION MODEL OF PHYSICAL GUARANTEE FOR HYDROPOWER PLANTS

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## Overview

The physical guarantee is one of the pillars of the Brazilian Interconnected System (SIN – *Sistema Interligado Nacional*) for hydraulic generation management and represents the maximum amount of energy that a plant can commit in its contracts over a year. On the other hand, each plant is given the flexibility to allocate and distribute its annual physical guarantee on a monthly basis, as long as the total value remains constant. This allows for a more adherent adjustment to its energy supply contracts and risk profile, and this process is known as the seasonalization of the physical guarantee. In this paper, we propose a physical guarantee seasonalization model using game theory tools and time series forecasting models in order to define an equilibrium price to guide the agents' allocation decisions. This approach also takes into account forecasts made available by regulatory agencies to the market and the experience of managers.

## Methods

A forecasting model for the system energy load was initially adjusted and fitted with a Seasonal Autoregressive Integrated Moving Average with exogenous regressors (SARIMAX) model. The exogenous variables included were dummies, which allow calibration of the moments of fall and recovery from systemic crises. The load forecast data was used to obtain the gross generation of the system, according to equation (1), which is used to obtain the apportionment factor of the generation surplus and shortages in the system among the agents.

$$\eta_j = \varepsilon_j - (\tau_j + \omega_j + \beta_j + \phi_j + s_j) \quad (1)$$

where  $\varepsilon$  is the energy load;  $\tau$  is the thermal generation;  $\omega$  is the wind generation;  $\beta$  is the biomass generation;  $\phi$  is the solar generation;  $s$  represents the generation of small hydroelectric plants; and  $\varpi$  represents the percentage of the generation of small hydroelectric that is assigned to the relocation energy mechanism.

The apportionment of a particular month is proportional to the physical guarantee allocated by each agent for that specific month. To obtain the optimization for seasonalization of physical guarantee, the seasonalization process was modeled as a game between the energy producers and concept of Nash Balance was used to find the best strategies.

The payoff ( $\pi$ ) for the game is defined according equation (2):

$$\pi_{ij} = (\varphi_{ij} \times GSF_j - c_i) \times PLD_j \times h_j \quad (2)$$

Where,  $\varphi_{ij}$  is the physical guarantee allocated by player  $i$  to period  $j$  and GSF is the system component that represents the system capacity to provide the physical guarantee allocated by agents to the period  $j$ . The GSF for month  $j$  is obtained from the division of the gross generation ( $\eta$ ) by the system seasonalized physical guarantee ( $\Gamma$ ), as shown in equation (3).

$$GSF_j = \frac{\eta_j}{\Gamma_j} \quad (3)$$

We show that there is a best strategy to be followed by the player, which is the one where the player seasonalize their physical guarantee ( $\varphi$ ) using the same proportion to the spot prices (PLD):

$$g_{ij} = G_i \times \frac{PLD_j}{\sum_{z=1}^{12}(PLD_z)} \quad (4)$$

Where  $G_i$  is the total physical guarantee of player  $i$ .

## Results

The results of the model development and application show that the Nash equilibrium will exist for the game and will be the one where the values of predicted energy spot prices for any month multiplied by the energy apportionment for that month will be the same. This conclusion derives from the non-arbitrage argument. A numerical application using actual data from a large player was performed to ensure the results reflect the strategies adopted by the players in the market and are consistent with the model predictions.

## Conclusions

A new approach using game theory was used to develop the model of seasonalization of physical guarantee of hydroelectric power generators in Brazil. This approach helps to clarify the dynamics underlying this complex process and indicates the best strategy for the players. The model developed can help agents optimize the results of the annual physical guarantee seasonalization process.

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