## A COBWEB MODEL FOR MULTIPHASE MARKETS

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### Overview

In this contribution we identify and study a class of markets which we call multiphase, characterized by a succession of market phases, where demand and/or supply conditions cyclically alternate with a precise and known periodicity. A typical example of such markets includes power exchanges. We introduce a new theoretical framework, based on the cobweb model with an adaptive multiphase expectation mechanism, to understand how the periodical nature affects the resulting dynamics. Both market characteristics and agents' expectations over market fundamentals play an essential role in determining the dynamic properties of the market equilibrium and introduce several dynamic behaviors which do not arise in a standard single phase cobweb model. Through numerical experiments, we show that such new multiphase framework and the resulting dynamics are both essential to reproduce price series which encompass the qualitative properties of those observed in real markets, in particular compared to real electricity price series.

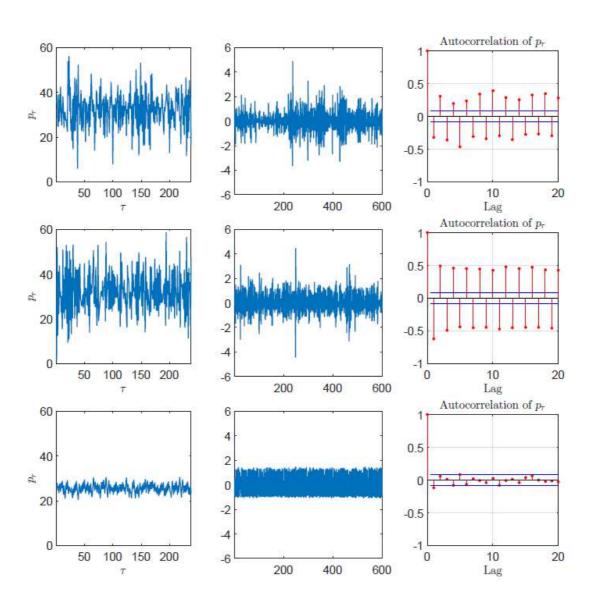
#### **Methods**

We propose a mathematical modelling based on the cobweb framework. To the best of our knowledge, the literature lacks a theoretical multiphase modelling for such kind of markets. We focus on a general, theoretical model able to analyze prototypical multiphase markets by means of both analytical and numerical investigations. This allows us to focus both on the demand/supply characteristics and on the agents' behaviour as fundamental drivers of market results. In this connection, since in actual examples we may observe stable demand/supply conditions over time as well as market environments intrinsically unstable, both market characteristics and agents' expectations over market fundamentals play an essential role in determining the dynamic properties of the market equilibrium. In particular, due to the complexity of multiphase markets and to the influence on dynamics of not predictable events, the assumption of agents' perfect foresight over prices cannot be retained and the way in which boundedly rational expectations are modeled becomes fundamental. Even if agents are not fully rational, we assume that they are aware of the cyclical nature of the market and we introduce an adaptive multiphase expectation mechanism. More precisely, agents form their expectations being aware of the cyclical recurrence of demand/supply functions and, as in classical adaptive expectations, they weight expected and realized prices through an expectation weight  $\omega$ .

## **Results**

We present the results obtained for a two-phases model: a peak and a off-peak phase. We find that the demand/supply characteristics and the agents' behavior are both fundamental drivers of market results. In a single phase setting driven by an adaptive expectation mechanism, such elements are represented by the reactivity of producers and consumers at the (unique) steady state equilibrium and by how much reliability the agents assign to the expectation error, or equivalently if they give more or less relevance to realized prices with respect to expected ones. In a multiphase setting the situation is much more complicated. Firstly, it is not possible to neglect the effect of the peculiar market structure on the agents' decisional mechanism. This is probably the most fundamental aspect characterizing the multiphase framework; agents can use and mix information coming from different past market phases, and consequently introduce mutual interdependence between the price dynamics of different market phases. Agents can again give more or less relevance to the realized expected prices with respect to the expected ones, but they can give different relevance to each phase prices, too. The complexity and richness of the possible dynamical scenarios is even more a consequence of a joint effect of the market structure and of the agents' behavior rule, which can not be completely disentangled. We report a typical scenario arising in a real multiphase market, namely the electricity market. Our goal is to check whether the theoretical approach we pursued (even in its simplest double phase setting) is able to reproduce several stylized aspects of the economic observables. In particular, it is wellknown that electricity prices exhibit highly erratic trajectories, which can not be described by a Gaussian random process. Typical features observed are nonnormality of distributions of returns, which are strongly leptokurtic and very volatile. In the first row of Figure 1 we report the price series and the corresponding returns, normalized by the volatility, of the illustrative example taken from the Italian electricity market. The resulting distribution is significantly non normal and leptokurtic (Kurt  $\approx 5.64$ ), with a large volatility  $\sigma \approx 51.3\%$ . In Figure 1 we also report the autocorrelation diagram of the price series. In the second row of Figure 1 we report a typical scenario arising from our simulations. As we can see, both the price series and the return distribution look very similar to their real market counterparts, with a strongly erratic behavior and frequent positive and negative spikes. In the third row of Figure 1 we present results of simulations using the standard (single-phase) Cobweb model. It is evident that it is not able to reproduce the dynamics of real world electricity prices.

Figure 1



# **Conclusions**

Comparing the simulated time series for a double phase model to the real price series of the electricity day-ahead market highlighted the importance of the multiphase framework for the emergence of stylized facts in the price series and the return distributions. Strongly erratic behavior, frequent positive and negative spikes and the significant leptokurtic deviation from normality of the resulting price distributions can be understood in terms of the boundedly rational expectation mechanism acting in a multiphase setting, as the single phase framework proves to be unsatisfactory.