Empirical Performance of Commodity Derivative Models

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Overview

The valuation of commodities contingent claims depends on the process assumed for the underlying asset. While the drift is important for futures pricing, the volatility is the price driver for options. In this paper we compare the empirical pricing performance of a model with constant volatility (CV) with one with a stochastic volatility specification (SV). These models are applied to oil, copper and gold getting consistent results for all three commodities. First, the CV model is clearly a better alternative to price futures contracts; not only it is simpler, but also it has smaller errors. However, if it is used to price options contracts the error could be considerable higher. Second, the longer the option maturity, the less relevant are the differences in pricing errors. Third, the higher complexity of the SV model is reflected in the, about 10 times, larger execution times. Fourth, the results of the SV model, applied for the first time to gold and copper, strongly suggest the presence of unspanned stochastic volatility components in all three commodities. Choosing the best model to implement in a real situation depends on the objectives pursued and on the tradeoffs between effort and precision. The results presented are then not only new, but also relevant from a practitioner point of view.

Methods

Kalman Filter and Maximum Likelihood on constant and stochastic volatility multifactor Brownian motions in non-arbitrage models for fitting oil futures and options are used

Results

-For Futures pricing CN models are more accurate and simpler to implement
-For Options pricing CV are more accurate but more complex to implement
-The longer the maturityof the options the less relevant is which model is used.

Conclusions

Prices of commodities contingent claims depend on the process assumed for the underlying asset. For futures, a good specification of the drift is very important, but for options, the volatility specification is crucial. Roughly speaking, commodity models in the literature can be classified in two types: those with a constant volatility and those with a stochastic volatility specification. In this paper, these two ways of dealing with volatility are contrasted in a way relevant to practitioners eliciting the trade-offs between the empirical performance and the implementation effort required for each model and commodity contract.

To make this comparative analysis, the Cortazar and Naranjo (2006) CN model to represent a constant volatility specification and the Trolle and Schwartz (2009) TS model as the stochastic volatility one, are chosen. Both models, specified with the same number of parameters (to make them comparable), are then applied to futures and options data for oil, copper and gold during different time periods. Pricing errors are calculated and execution times measured.

Results for all commodities are, in general, consistent. First, for futures pricing it is clearly better to use the CN model, because not only it is simpler but also errors are smaller. Also the higher the number of risk factors in the model, the better. Second, options pricing errors are considerably higher using the CN model increasing at the most by a factor of 6. Third, the longer the option maturity, the less relevant is the difference in pricing errors. For long maturity contracts the error difference is small. Fourth, the TS is much more complex to implement and its execution times are about 10 times higher. Fifth, our results of implementing the TS model for copper and gold are consistent with unspanned stochastic volatility for both commodities.

Results presented in this paper are new and relevant for practitioners. Up to now it is, to our knowledge, the first work to empirically test the pricing performance, using futures and options contracts, of stochastic volatility models against constant volatility benchmarks for oil, copper and gold. Also it is the first to apply the TS model to copper and gold markets. Choosing the best model to implement in a real situation depends on the objectives pursued and in the tradeoffs between effort and precision.

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